Mechanics of Structure Genome:

Fill the Gap between Materials Genome and Structural Analysis

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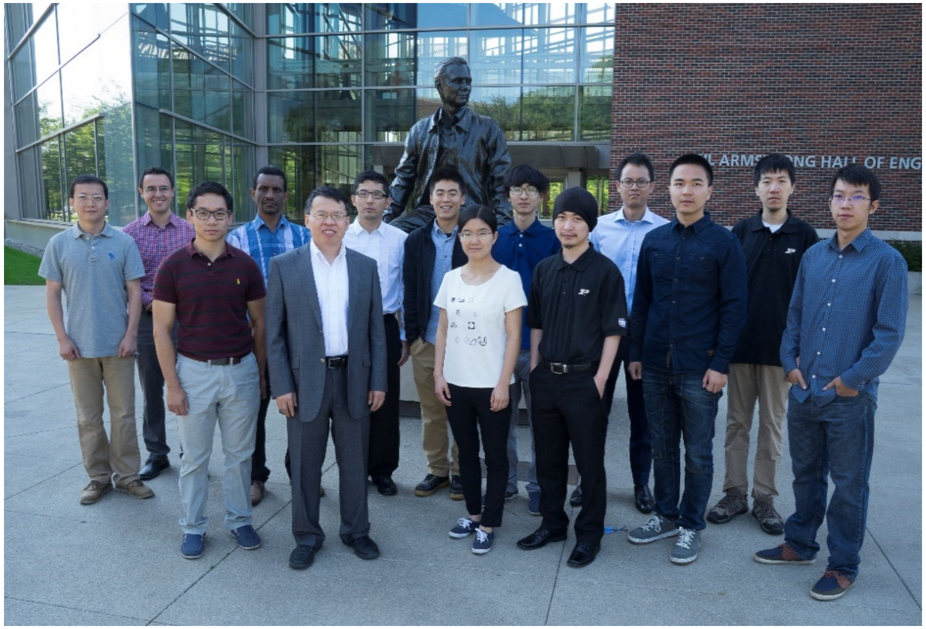
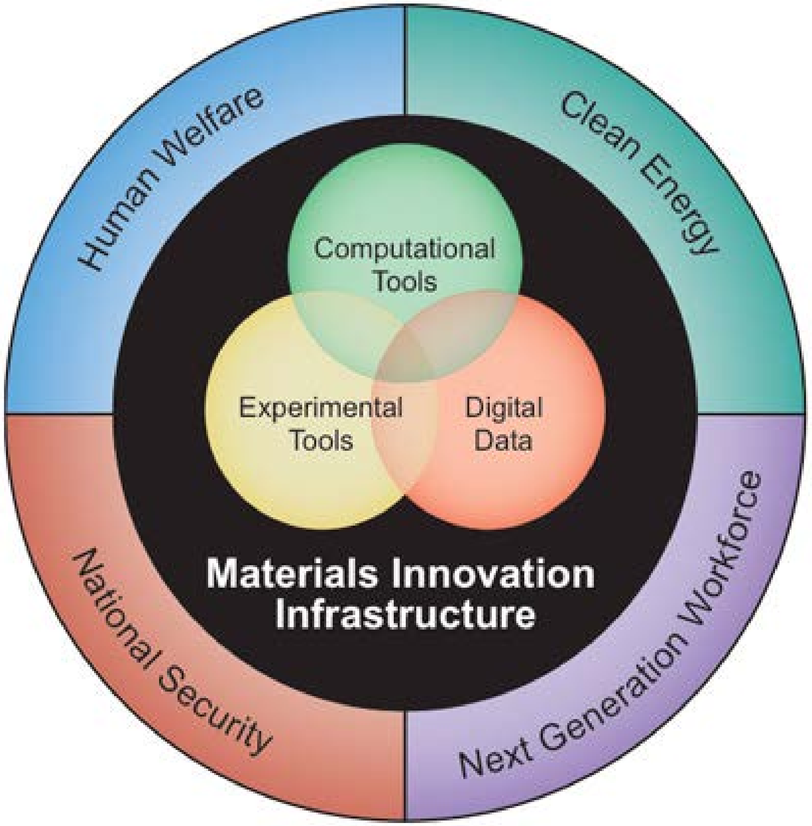
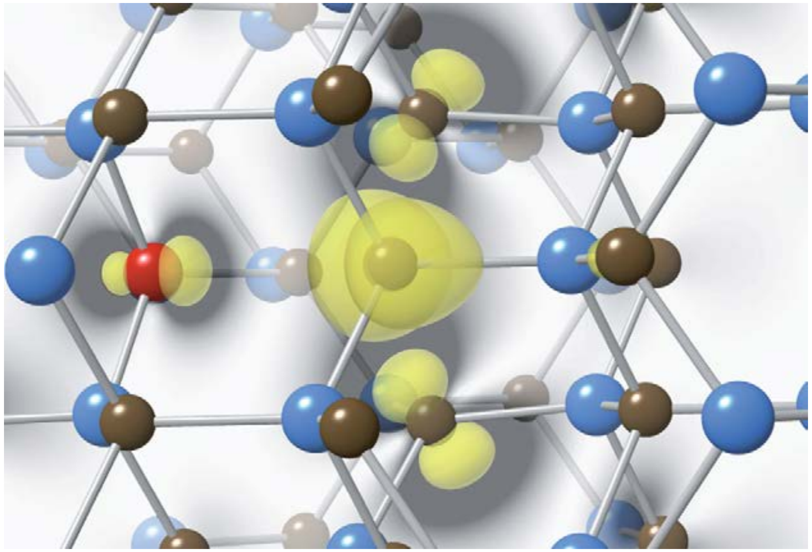
Vision/Mission/Values

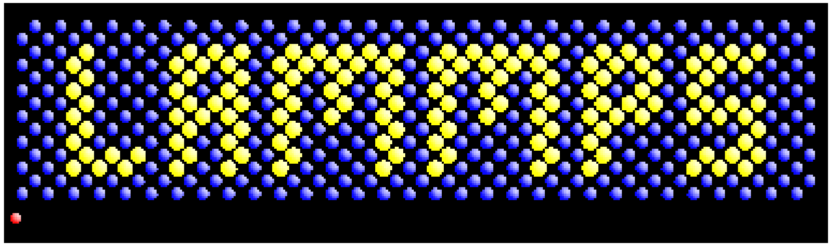
- **Vision**: Design, manufacture, and certify advanced structures and materials by analysis.
- **Mission**: To advance predictive capabilities for advanced structures and materials and to train students with analysis fundamentals and job-ready skills.
- **Professional Values**
  - We pursue **truth** because truth can set us free.
  - We seek **unity** to systematically handle diversities.
  - We strike for **balance** between practicality and rigor.
  - We embrace **humble boldness** to learn from others and remain true to scholarship.



Materials Genome Initiative

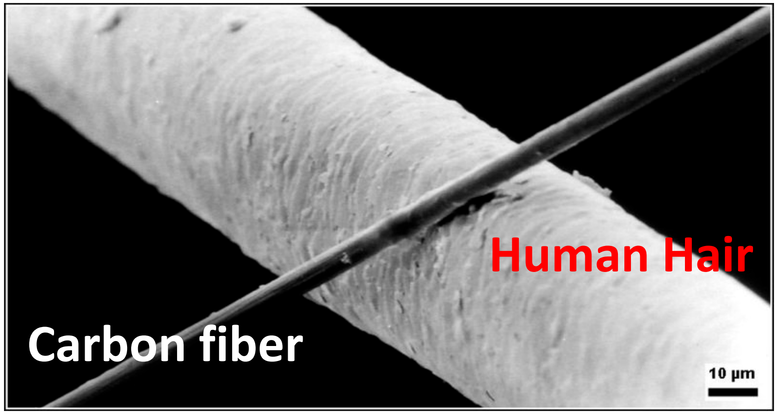


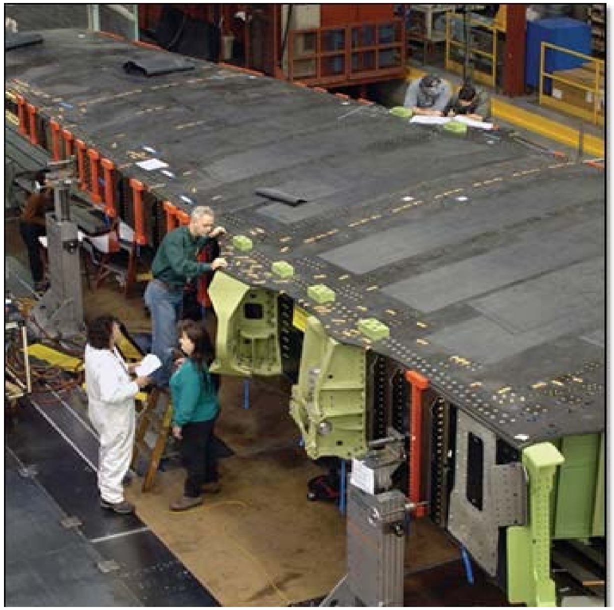




MGI deliverables: properties, allowables, failure criteria for constituents (fiber, matrix), interfaces

The Challenge: Multiple Scales

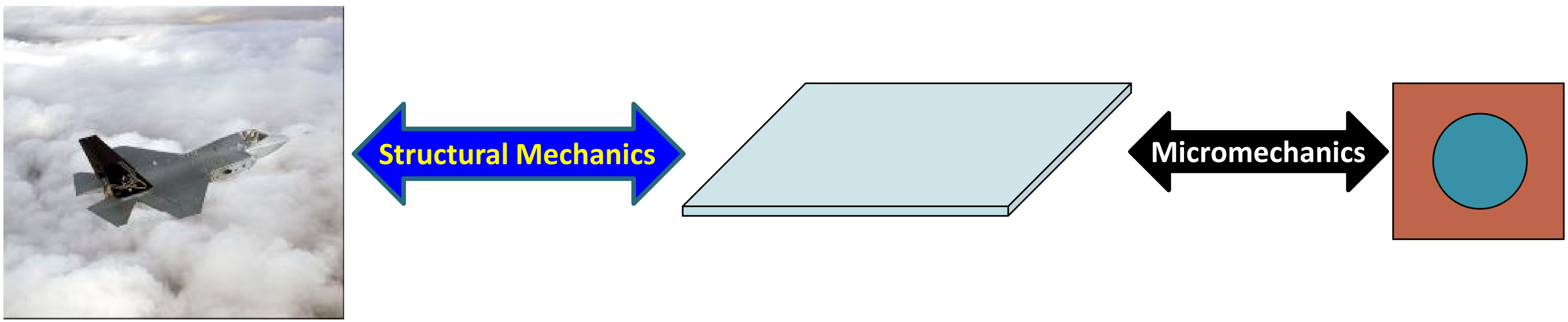


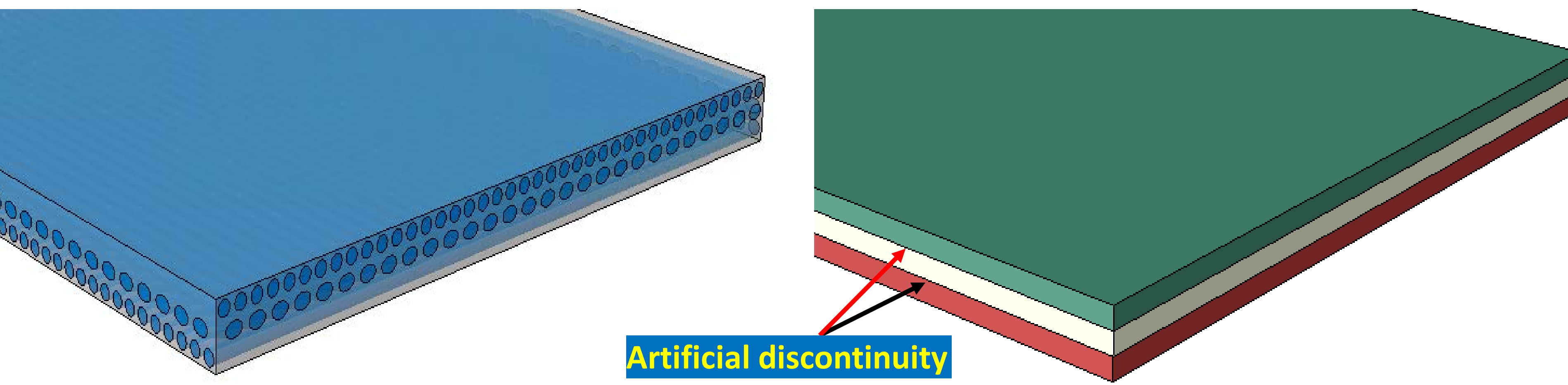


1 mm^3 material block
~ **20 Million DOFs**

Bottom-up Multiscale Modeling

Structure	Meso	Micro
10^{1} m	10^{-3} m	10^{-6} m





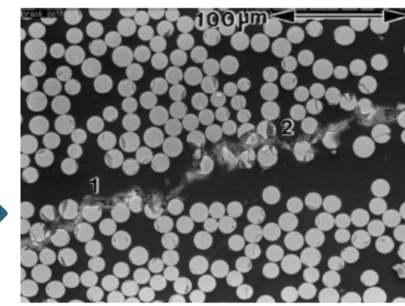
Top-Down Multiscale Modeling

Structural Analysis — 10^{1} m

Microstructure — 10^{-6} m

Mechanics of Structure Genome

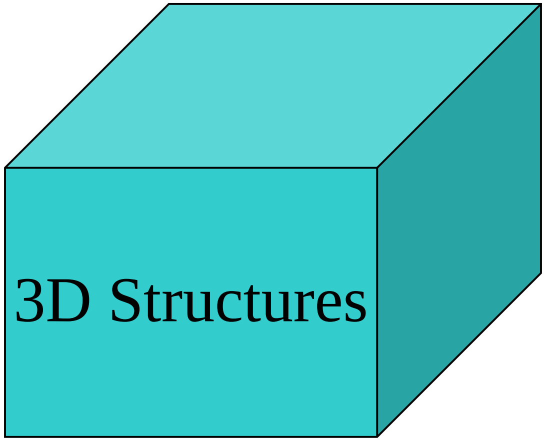
Minimize Information Loss

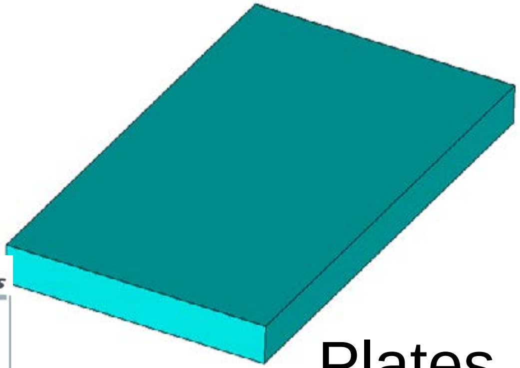


For since the fabric of the universe is most perfect and the work of a most wise creator, nothing at all takes place in the universe in which some rule of the maximum or minimum does not appear.

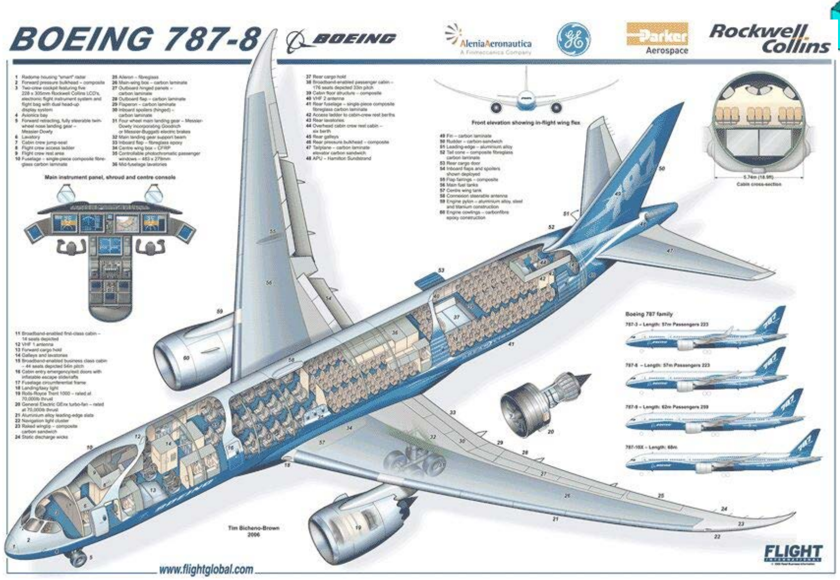
-- Leonhard Euler

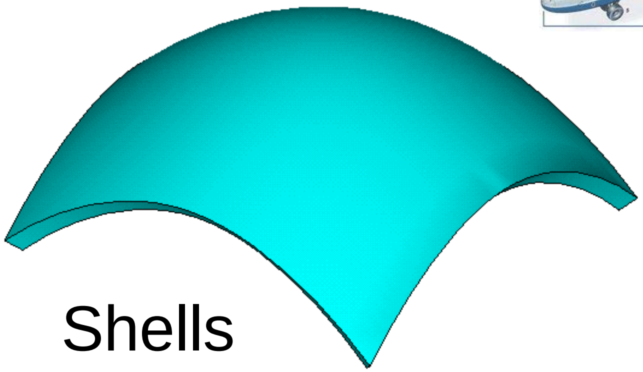
Typical Structural Components



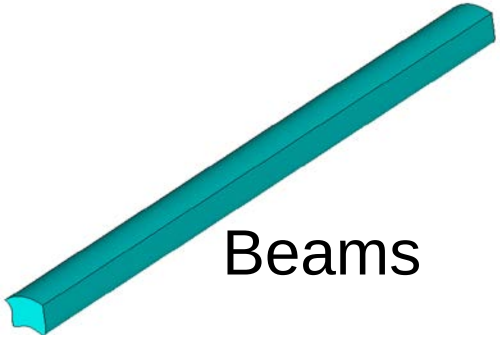


Plates

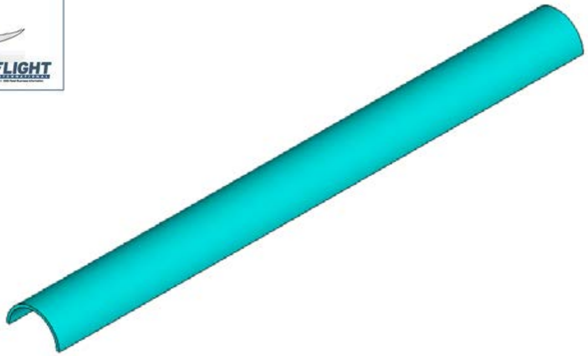




Shells

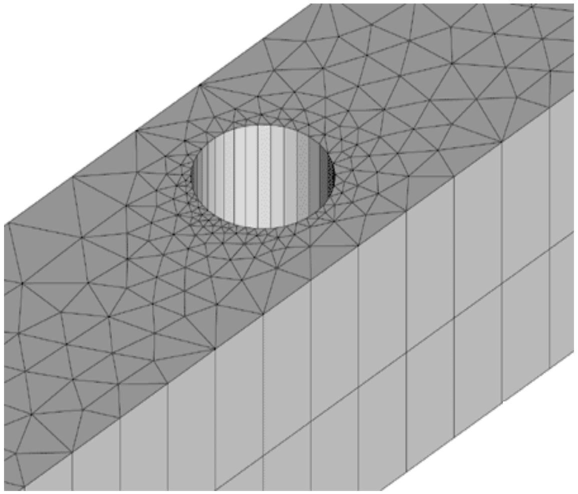


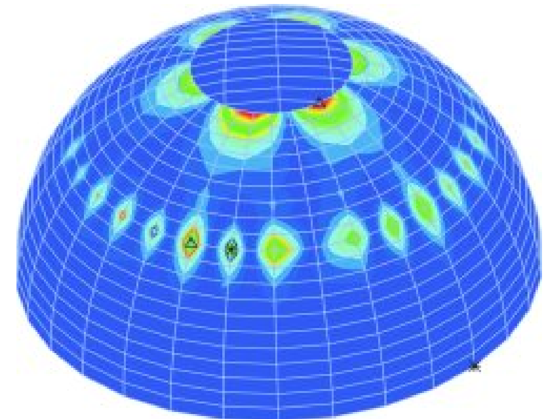
Beams

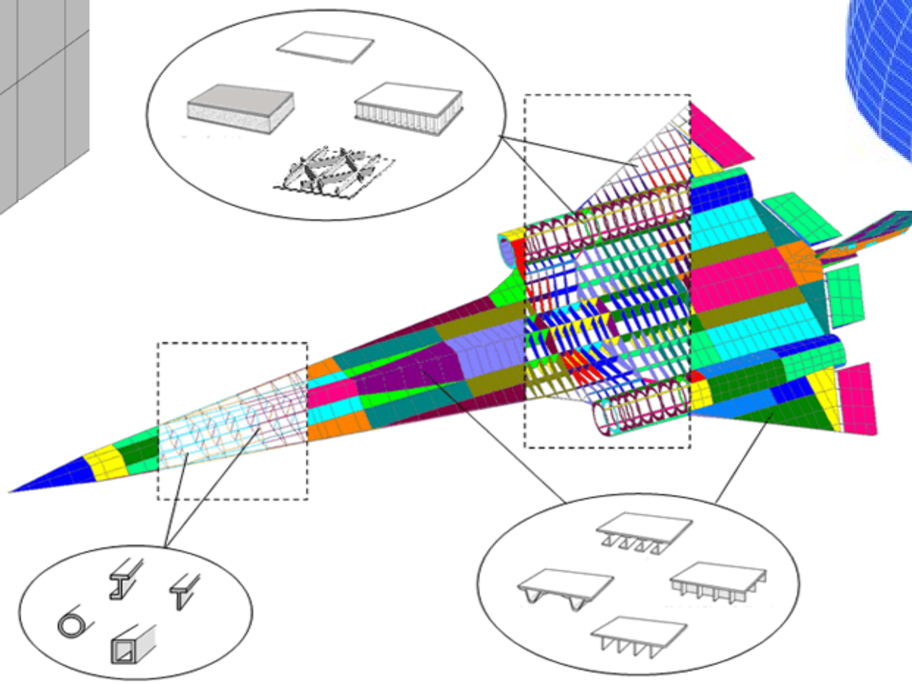


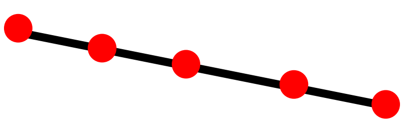
Thin-walled beams

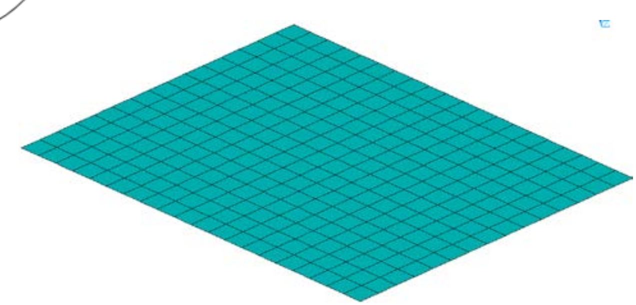
Structural Analyses







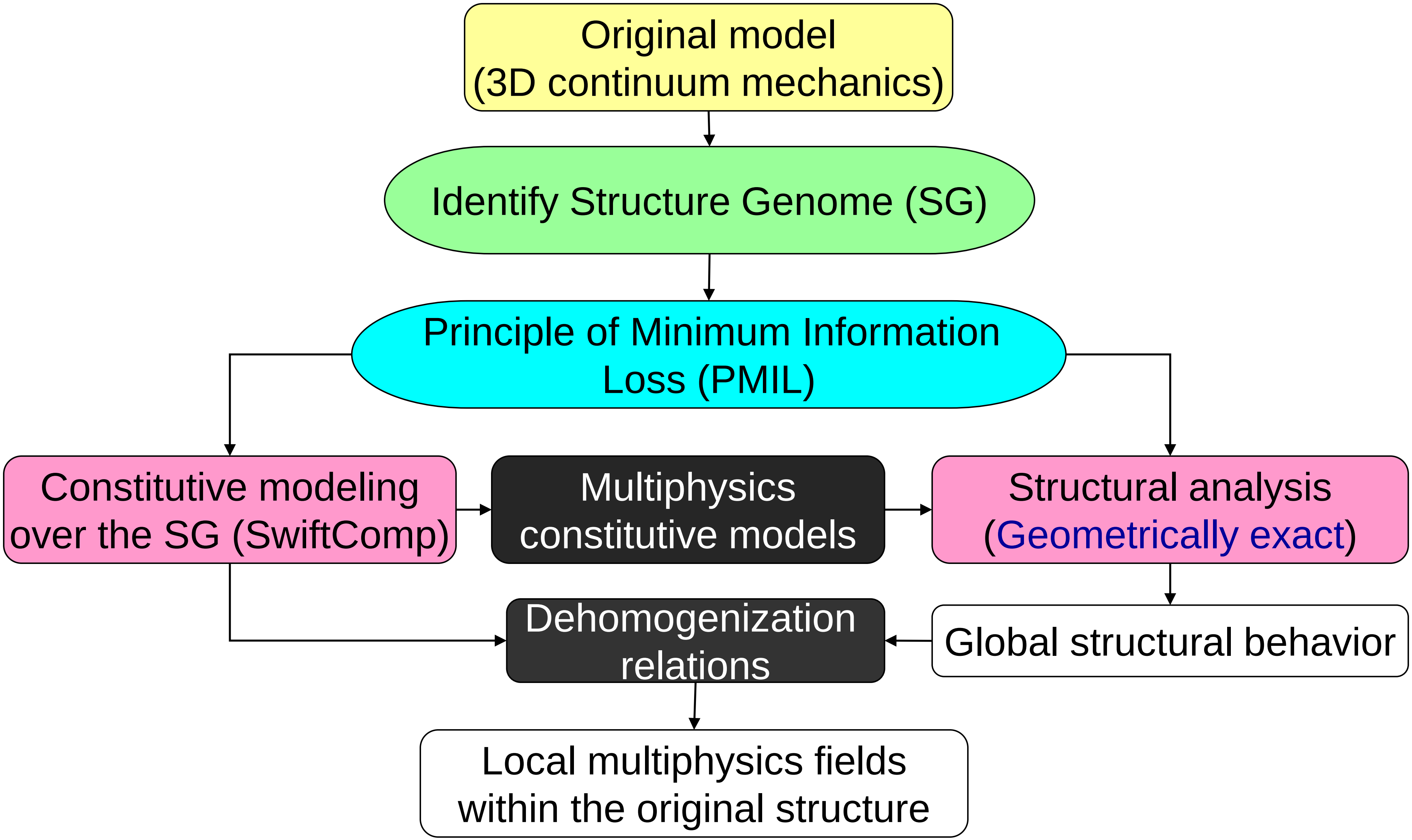




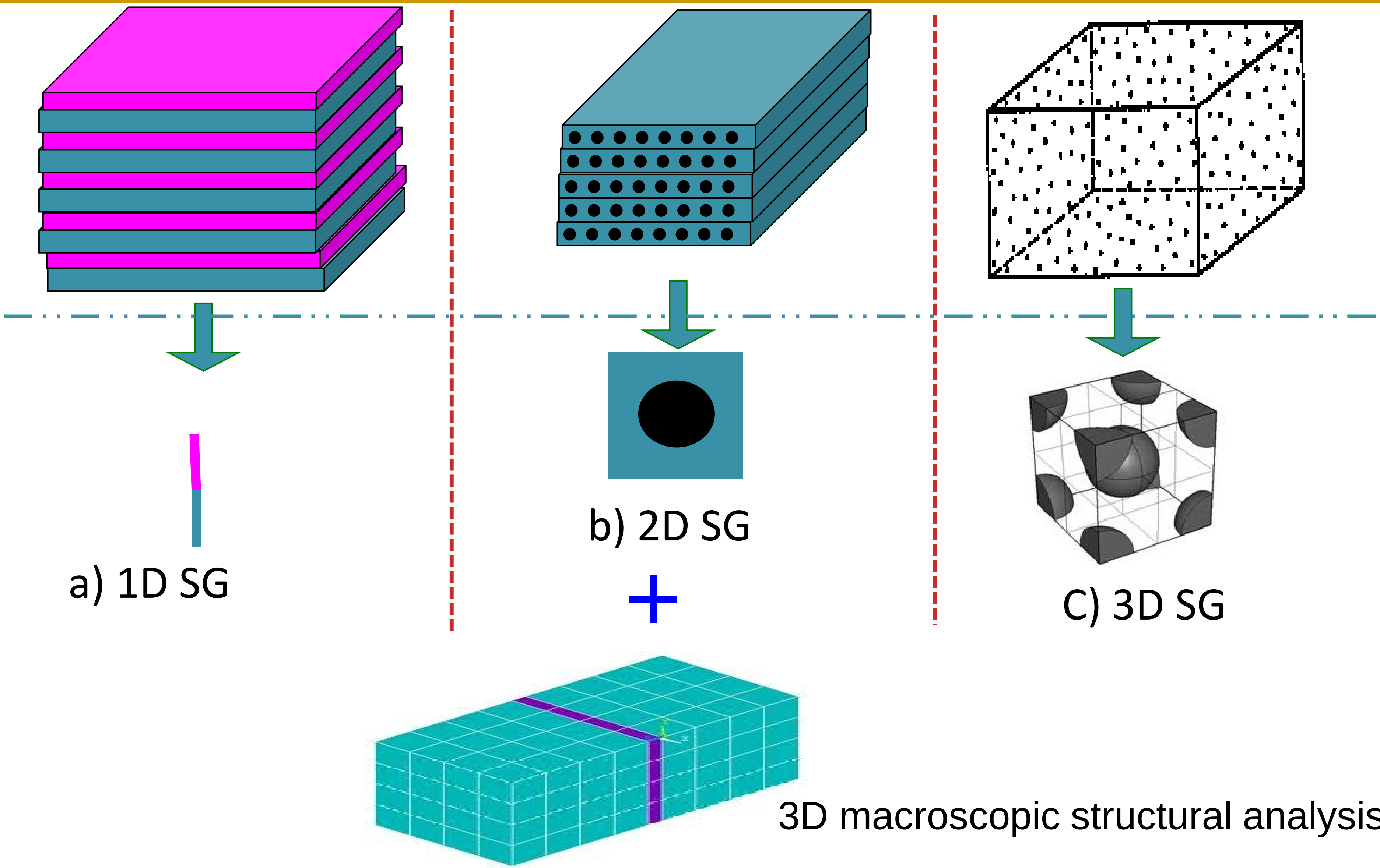
Structural Models

	Kinematics	Kinetics	Constitutive Relations
3D	$\varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i})$	$\sigma_{ij,j} + f_i = 0$	$\begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{23} \\ \sigma_{13} \\ \sigma_{12} \end{Bmatrix} = \begin{bmatrix} c_{11} & c_{12} & c_{13} & c_{14} & c_{15} & c_{16} \\ c_{12} & c_{22} & c_{23} & c_{24} & c_{25} & c_{26} \\ c_{13} & c_{23} & c_{33} & c_{34} & c_{35} & c_{36} \\ c_{14} & c_{24} & c_{34} & c_{44} & c_{45} & c_{46} \\ c_{15} & c_{25} & c_{35} & c_{45} & c_{55} & c_{56} \\ c_{16} & c_{26} & c_{36} & c_{46} & c_{56} & c_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ 2\varepsilon_{23} \\ 2\varepsilon_{13} \\ 2\varepsilon_{12} \end{Bmatrix}$
2D (Plate/ shell)	$\epsilon_{\alpha\beta} = \frac{1}{2}(\bar{u}_{\alpha,\beta} + \bar{u}_{\beta,\alpha})$ $\kappa_{\alpha\beta} = -\bar{u}_{3,\alpha\beta}$	$N_{\alpha\beta,\beta} + p_\alpha = 0$ $(M_{\alpha\beta,\beta} + e_{\alpha\beta} q_\beta)_{,\alpha} + p_3 = 0$	$\begin{Bmatrix} N \\ M \end{Bmatrix} = \begin{bmatrix} A & B \\ B^T & D \end{bmatrix} \begin{Bmatrix} \epsilon \\ \kappa \end{Bmatrix}$
1D (beam)	$\gamma_{11} = \bar{u}_1'$ $\kappa_1 = \Phi_1'$ $\kappa_2 = -\bar{u}_3''$ $\kappa_3 = \bar{u}_2''$	$\frac{dF_1}{dx_1} + p_1 = 0$ $\frac{dM_1}{dx_1} + q_1 = 0$ $\frac{d^2 M_2}{dx_1^2} + p_3 + \frac{dq_2}{dx_1} = 0$ $\frac{d^2 M_3}{dx_1^2} - p_2 + \frac{dq_3}{dx_1} = 0$	$\begin{Bmatrix} F_1 \\ M_1 \\ M_2 \\ M_3 \end{Bmatrix} = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{12} & S_{22} & S_{23} & S_{24} \\ S_{13} & S_{23} & S_{33} & S_{34} \\ S_{14} & S_{24} & S_{34} & S_{44} \end{bmatrix} \begin{Bmatrix} \gamma_{11} \\ \kappa_1 \\ \kappa_2 \\ \kappa_3 \end{Bmatrix}$

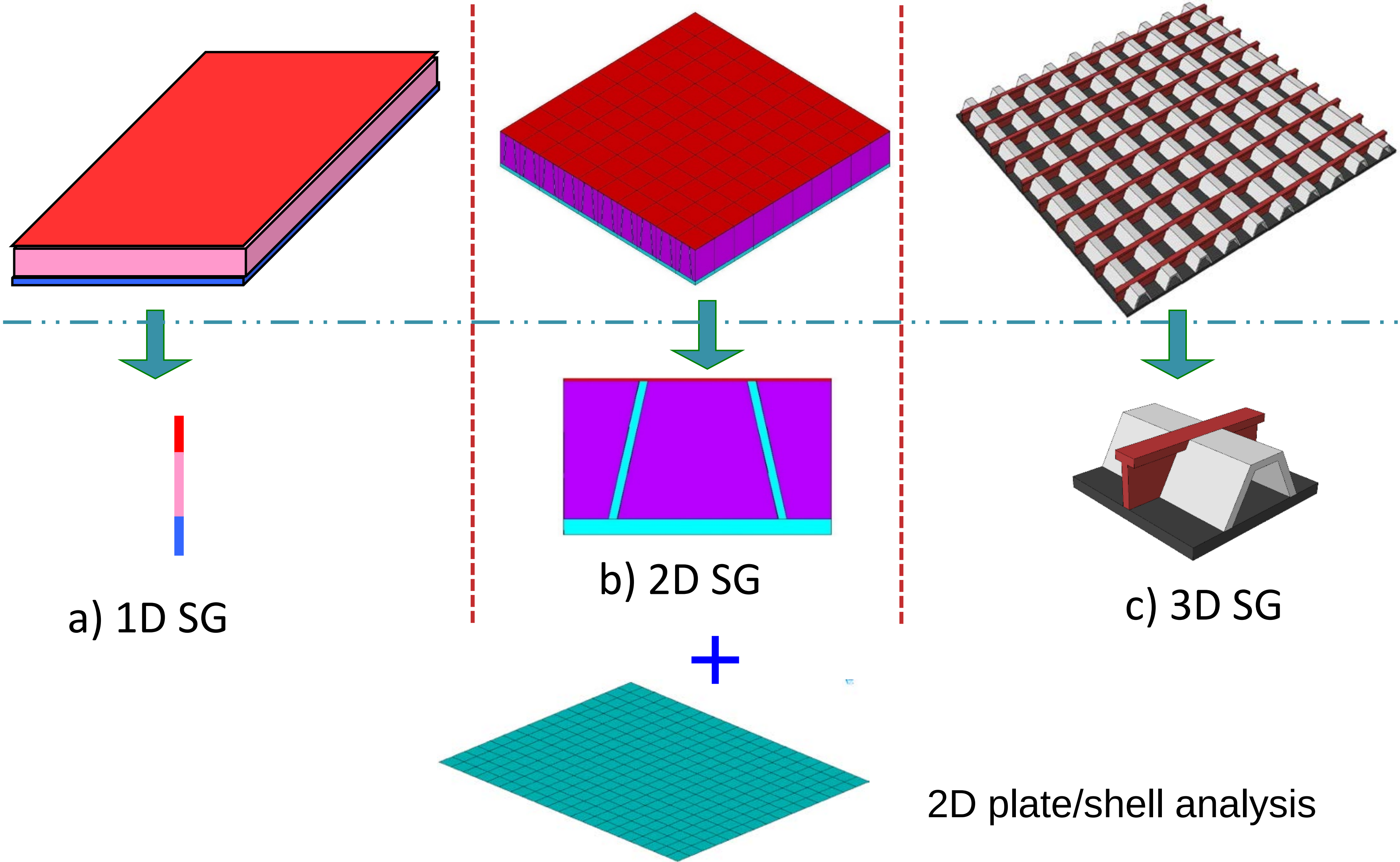
Mechanics of Structure Genome



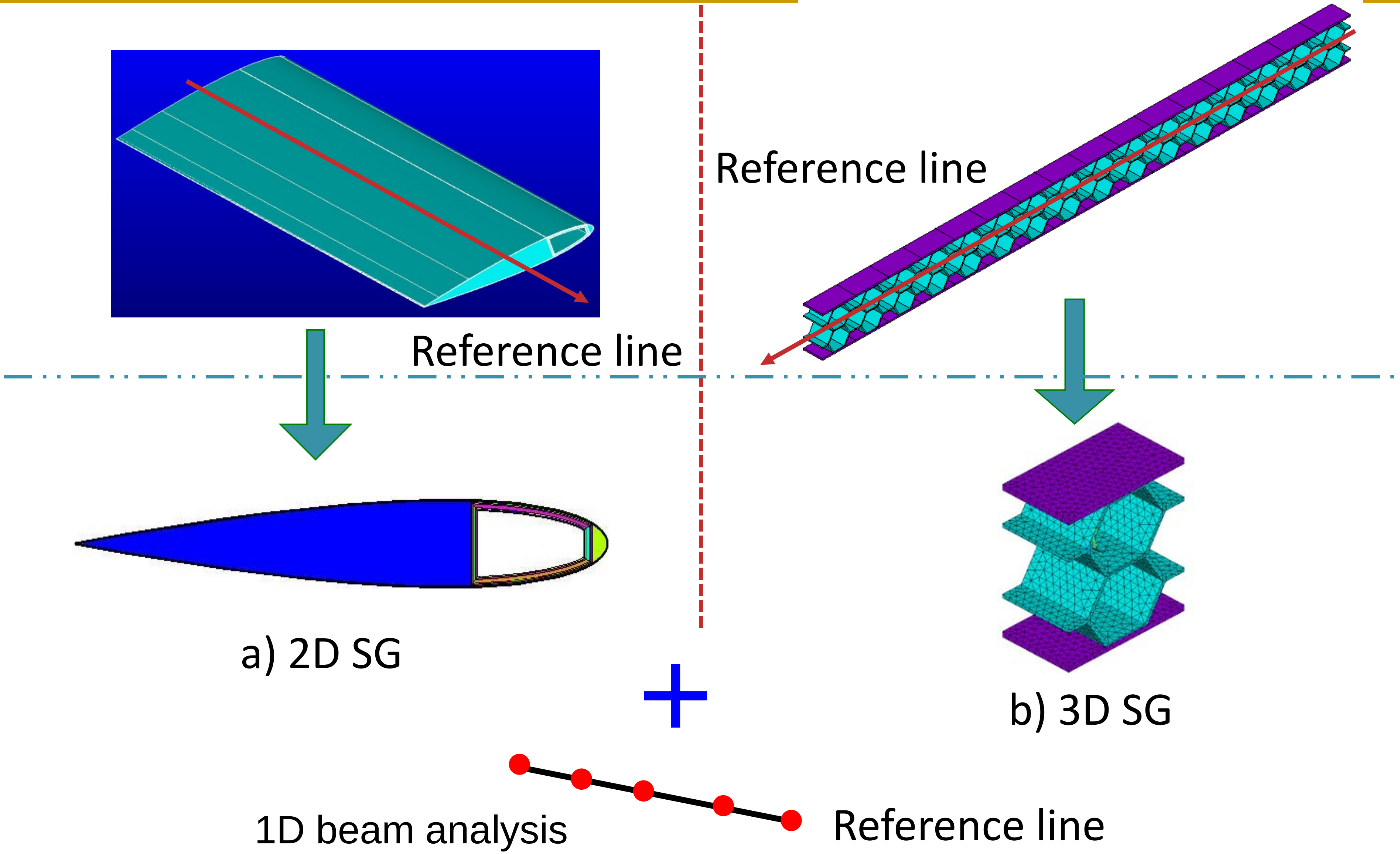
SG for 3D Structures



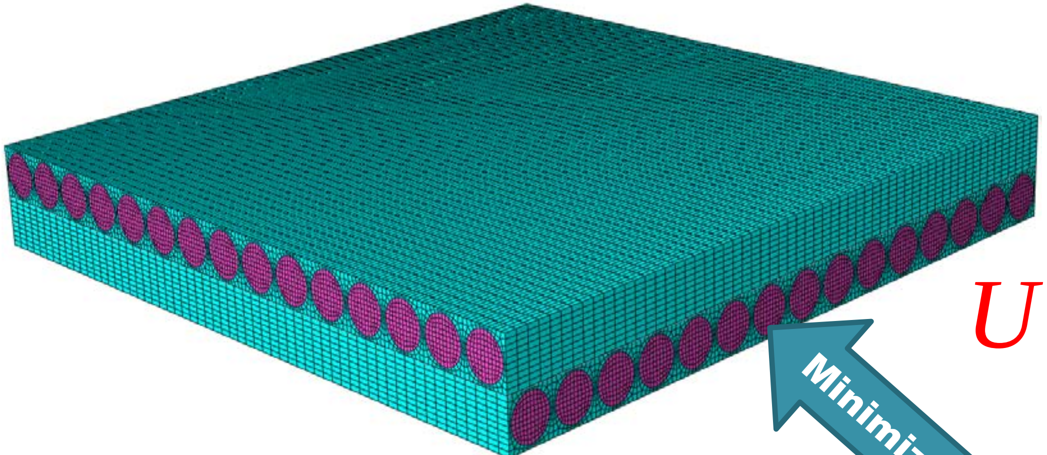
SG for Panels (Plates/Shells)



SG for Beam-like Structures



MSG for CLPT



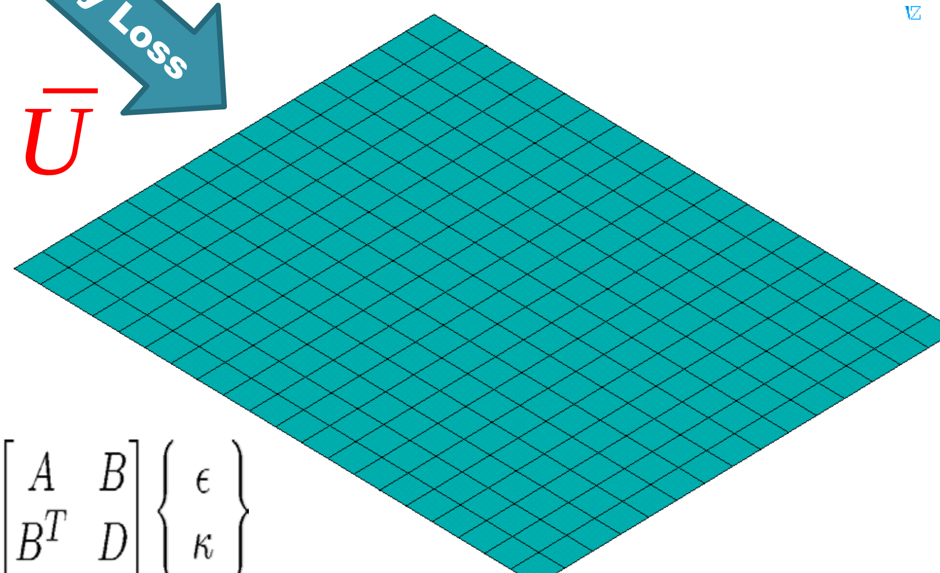
$$\sigma_{ij,j} + f_i = 0$$

$$\varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i})$$

$$\begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{23} \\ \sigma_{13} \\ \sigma_{12} \end{Bmatrix} = \begin{bmatrix} c_{11} & c_{12} & c_{13} & c_{14} & c_{15} & c_{16} \\ c_{12} & c_{22} & c_{23} & c_{24} & c_{25} & c_{26} \\ c_{13} & c_{23} & c_{33} & c_{34} & c_{35} & c_{36} \\ c_{14} & c_{24} & c_{34} & c_{44} & c_{45} & c_{46} \\ c_{15} & c_{25} & c_{35} & c_{45} & c_{55} & c_{56} \\ c_{16} & c_{26} & c_{36} & c_{46} & c_{56} & c_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ 2\varepsilon_{23} \\ 2\varepsilon_{13} \\ 2\varepsilon_{12} \end{Bmatrix}$$

U

$\bar{U}$



$$\epsilon_{\alpha\beta} = \frac{1}{2}(\bar{u}_{\alpha,\beta} + \bar{u}_{\beta,\alpha})$$

$$\kappa_{\alpha\beta} = -\bar{u}_{3,\alpha\beta}$$

$$N_{\alpha\beta,\beta} + p_\alpha = 0$$

$$(M_{\alpha\beta,\beta} + e_{\alpha\beta}q_\beta)_{,\alpha} + p_3 = 0$$

$$\begin{Bmatrix} N \\ M \end{Bmatrix} = \begin{bmatrix} A & B \\ B^T & D \end{bmatrix} \begin{Bmatrix} \epsilon \\ \kappa \end{Bmatrix}$$

MSG for CLPT (Cont.)

- Express kinematics of the original model in terms of that of the macroscopic model and unknown warping/fluctuating functions

$$u_i = u_i(\bar{u}, \bar{u}_{,\alpha}, w_i)$$

$$\varepsilon_{ij} = \varepsilon_{ij}(\bar{\varepsilon}_{\alpha\beta}, \kappa_{\alpha\beta}, w_{i,j})$$

- Define kinematics of the macroscopic model in terms of the original model

MSG for CLPT (Cont.)

- Express the energy of the original model

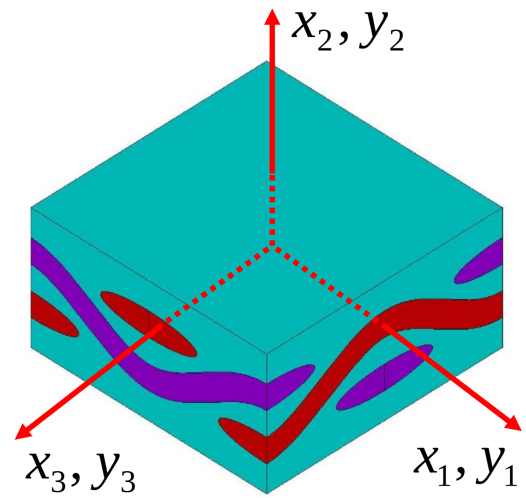
$$U = U(\varepsilon_{ij}) = U(\overline{\varepsilon}_{\alpha\beta}, \kappa_{\alpha\beta}, w_{i,j})$$

- Minimize the energy to solve fluctuating functions

$$\min_{w_i} U(\overline{\varepsilon}_{\alpha\beta}, \kappa_{\alpha\beta}, w_{i,j}) = \overline{U}(\overline{\varepsilon}_{\alpha\beta}, \kappa_{\alpha\beta})$$

MSG for Aperiodic Materials

- Kinematics



  - Displacement

$$u_i(\mathbf{x};\mathbf{y}) = v_i(\mathbf{x}) + \varepsilon \chi_i(\mathbf{x};\mathbf{y})$$

  - Strain

$$\epsilon_{ij}(\mathbf{x};\mathbf{y}) = \frac{1}{2}\left[\frac{\partial u_i(\mathbf{x};\mathbf{y})}{\partial x_j} + \frac{\partial u_j(\mathbf{x};\mathbf{y})}{\partial x_i}\right] = \bar{\epsilon}_{ij} + \chi_{(i|j)} + \varepsilon \chi_{(i,j)}$$

  - Kinematic equivalency constraints: average displacement and strain

$$v_i = \frac{1}{\Omega}\int_\Omega u_i \, \mathrm{d}\Omega \equiv \langle u_i \rangle \Rightarrow \langle \chi_i \rangle = 0$$

$$\bar{\epsilon}_{ij} \equiv \langle \epsilon_{ij} \rangle \Rightarrow \left\langle \chi_{(i|j)} \right\rangle = 0$$

MSG for Aperiodic Materials

- Minimize energy discrepancy between the deformed heterogeneous material and the homogenized material

$$J = \Pi_{Micro} - \Pi_{Macro} - \lambda_{kl}\langle \chi_{(k|l)} \rangle - \eta_i \langle \chi_i \rangle \qquad \delta J = 0$$

$$\Pi_{Micro} = \langle C_{ijkl}\epsilon_{ij}\epsilon_{kl} \rangle = \langle C_{ijkl}\left(\bar{\epsilon}_{ij} + \chi_{(i|j)}\right)\left(\bar{\epsilon}_{kl} + \chi_{(k|l)}\right)\rangle \qquad \Pi_{Macro} = \langle \bar{C}_{ijkl}\bar{\epsilon}_{ij}\bar{\epsilon}_{kl} \rangle$$

- Finite Element Implementation

$$\chi(x_i; y_j) = S(y_j)V(x_i)$$

$$J = \frac{1}{2}\left(V^T E V + 2V^T D_{h\epsilon}\bar{\epsilon} + \bar{\epsilon}^T D_{\epsilon\epsilon}\bar{\epsilon}\right) - \frac{1}{2}V^{\mathrm{T}}\bar{D}^* V - \lambda^{\mathrm{T}} D_{h\lambda}^{\mathrm{T}} V$$

$$E = \left\langle (\Gamma_h S)^T D (\Gamma_h S) \right\rangle \quad D_{h\epsilon} = \left\langle (\Gamma_h S)^T D \right\rangle \quad D_{\epsilon\epsilon} = \langle D \rangle \quad D_{h\lambda} = \langle \Gamma_h S \rangle^{\mathrm{T}}$$

MSG for Aperiodic Materials

- The solution

$$V = V_0 \bar{\epsilon} \qquad \chi = S V_0 \bar{\epsilon}$$

- Homogenized energy

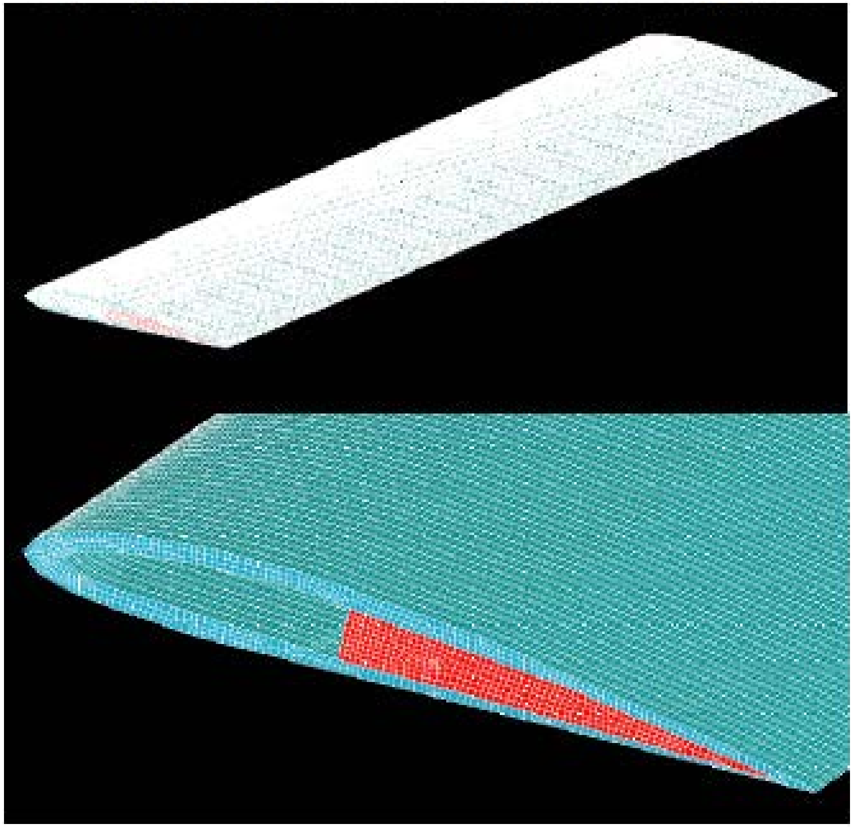
$$U = \frac{1}{2} \bar{\epsilon}^{\mathrm{T}} \left(D_{\epsilon\epsilon} + 2 V_0^{\mathrm{T}} D_{h\epsilon} + V_0^T E V_0 \right) \bar{\epsilon} \equiv \frac{\Omega}{2} \bar{\epsilon}^{\mathrm{T}} \bar{D} \bar{\epsilon}$$

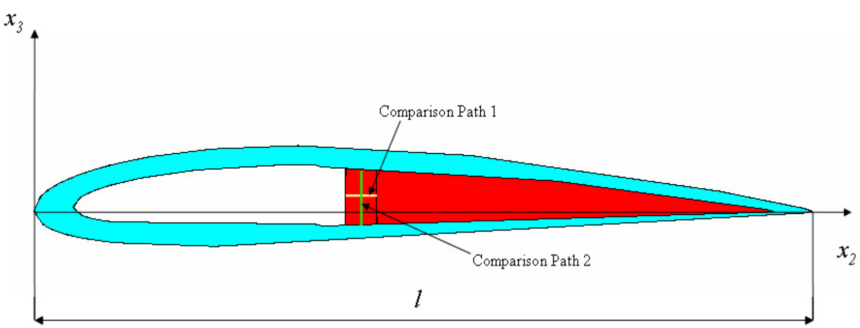
- Dehomogenization relations

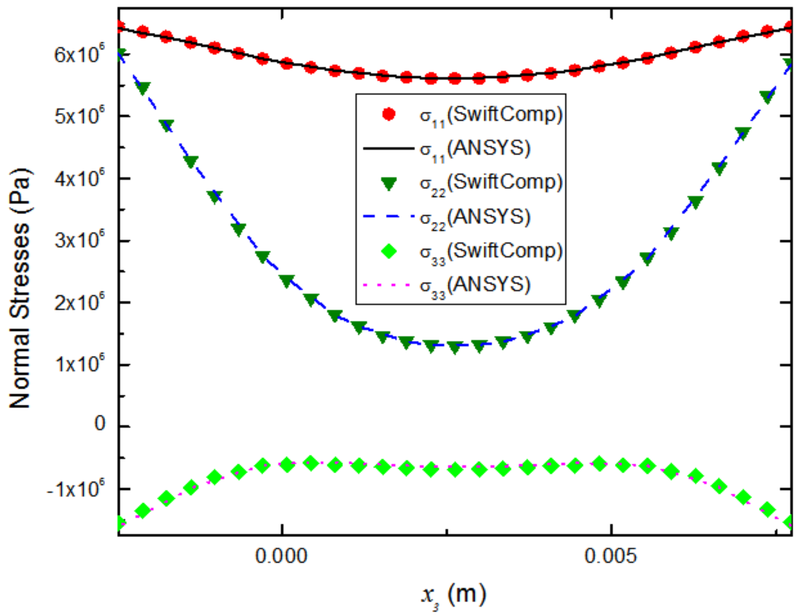
$$\boldsymbol{u} = \boldsymbol{v} + \begin{bmatrix} \frac{\partial v_1}{\partial x_1} & \frac{\partial v_1}{\partial x_2} & \frac{\partial v_1}{\partial x_3} \\ \frac{\partial v_2}{\partial x_1} & \frac{\partial v_2}{\partial x_2} & \frac{\partial v_2}{\partial x_3} \\ \frac{\partial v_3}{\partial x_1} & \frac{\partial v_3}{\partial x_2} & \frac{\partial v_3}{\partial x_3} \end{bmatrix} \begin{Bmatrix} y_1 \\ y_2 \\ y_3 \end{Bmatrix} + S V_0 \bar{\epsilon}$$

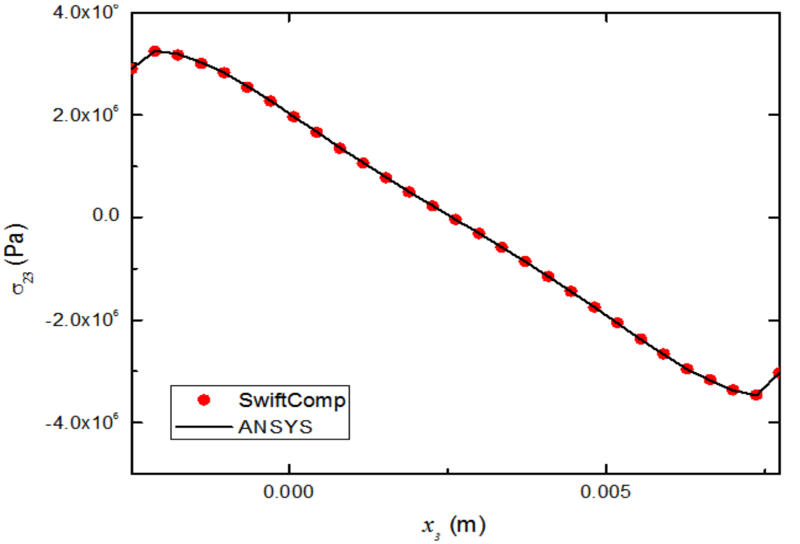
$$\epsilon = \bar{\epsilon} + \Gamma_h S V_0 \bar{\epsilon} \qquad \sigma = D \epsilon.$$

Realistic Rotor Blade



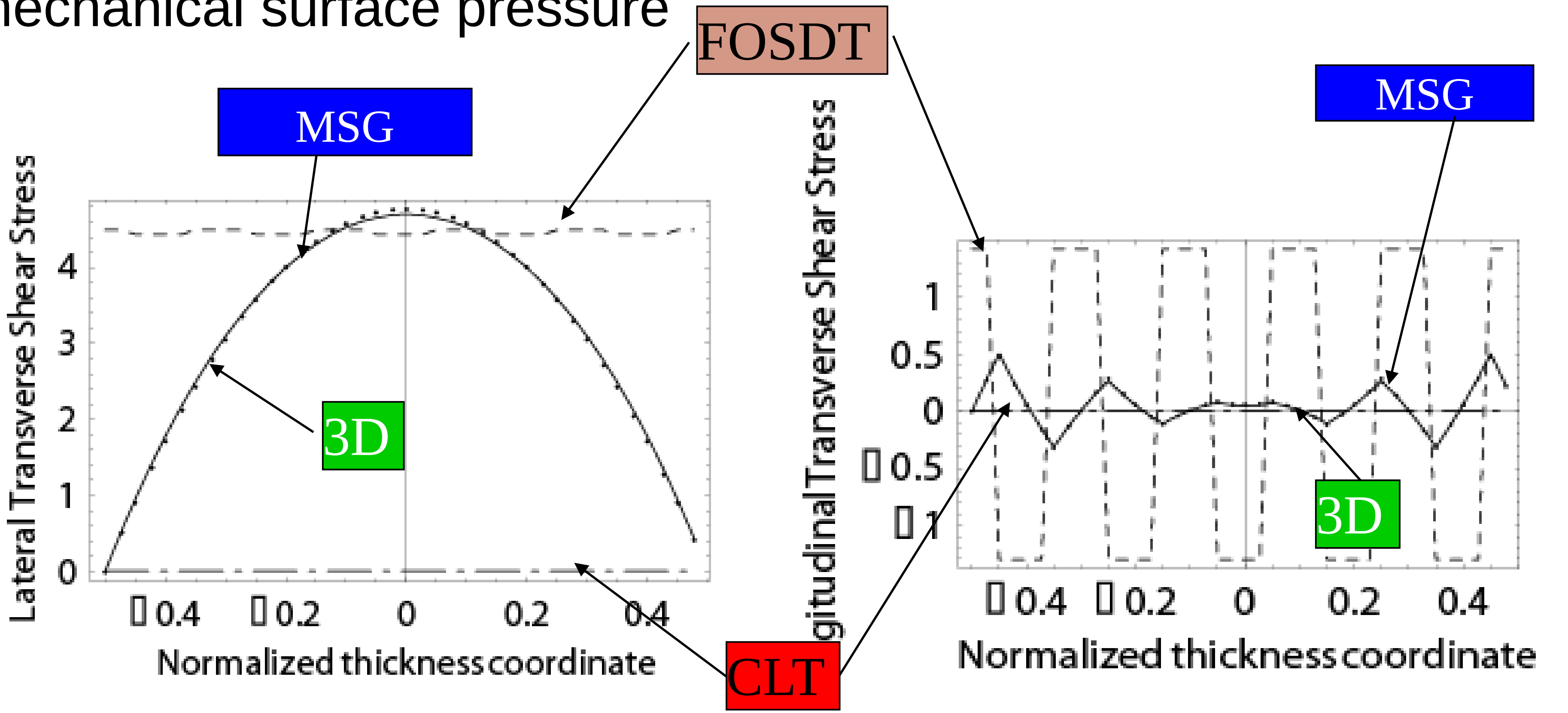






3D FEA Accuracy with FOSDT Cost

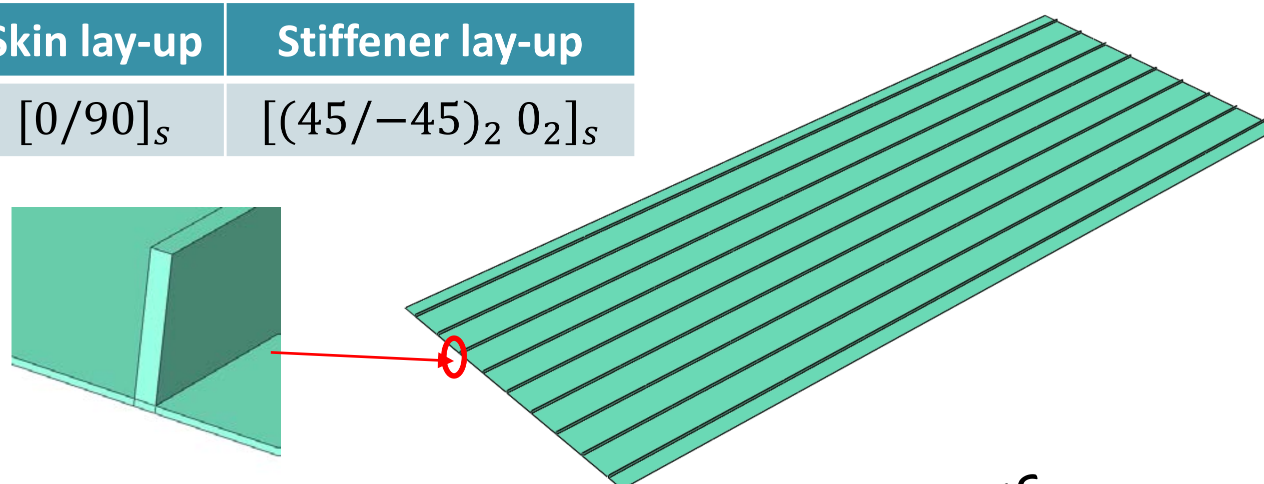
A 20-layer plate $[30^0/-30^0/-30^0/30^0]_5$ under a sinusoidal mechanical surface pressure



Achieving 3D elasticity accuracy at efficiency of FOSDT (Reissner-MindlinTheory)

Buckling Analysis of Stiffened Composite Panels

Skin lay-up	Stiffener lay-up
$[0/90]_s$	$[(45/-45)_2\ 0_2]_s$



Constitutive modeling using SwiftComp

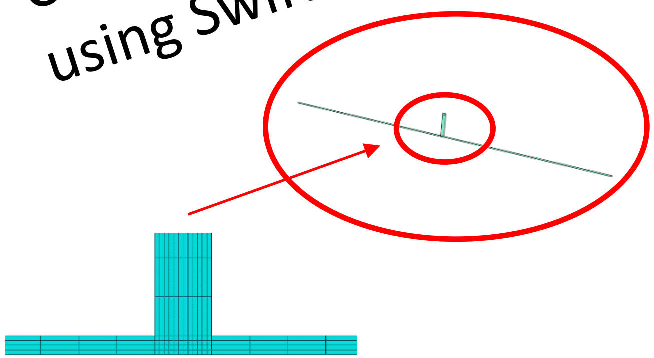
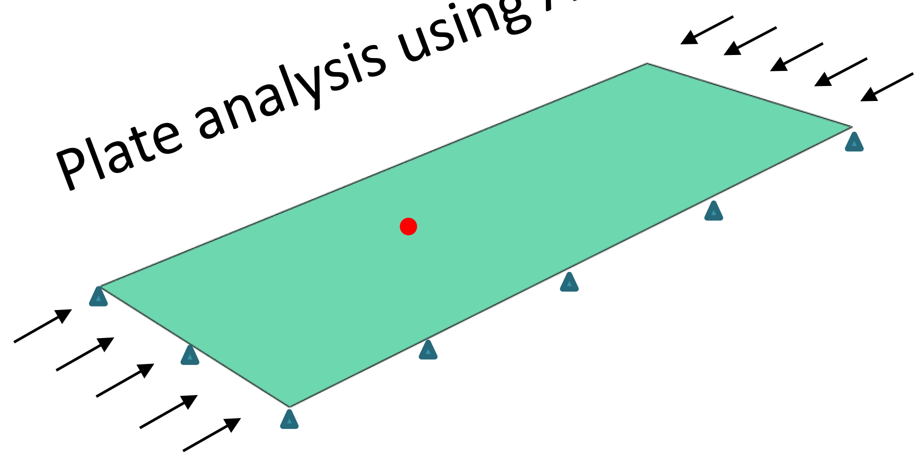


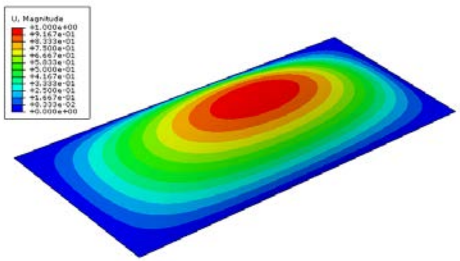
Plate analysis using ABAQUS



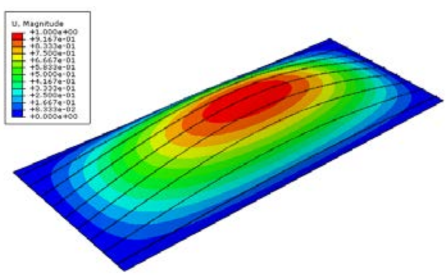
Buckling Analysis of Stiffened Composite Panels

1st mode

SwiftComp based plate

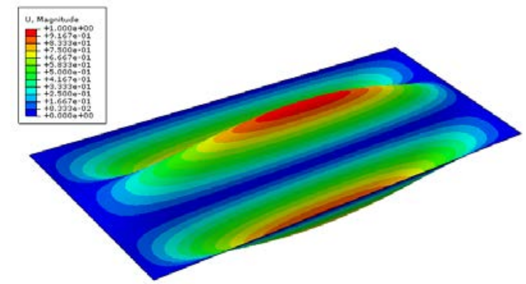


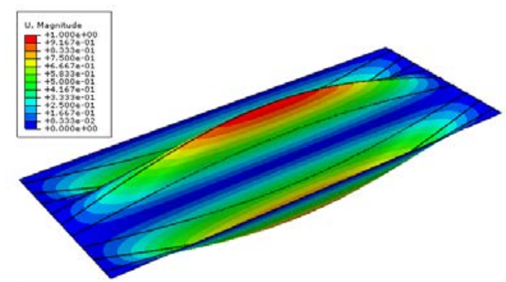
3D FEA



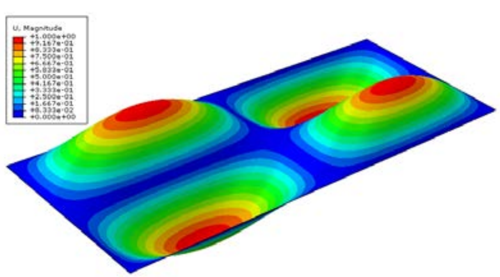
Half wave number	MSG-based plate analysis	3D FEA
m=1 n=1	930 (0.11%)	931
m=1 n=2	1799 (0.55%)	1809
m=2 n=1	3348 (1.42%)	3301
m=2 n=2	3743 (1.19%)	3699
m=2 n=3	4836 (0.71%)	4802
m=1 n=3	5009 (0.50%)	5034

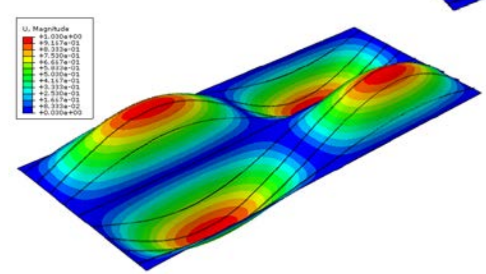
6th mode





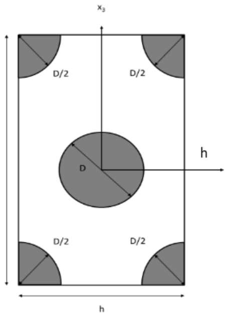
4th mode

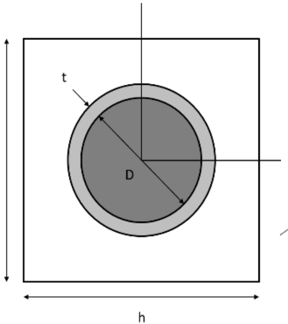


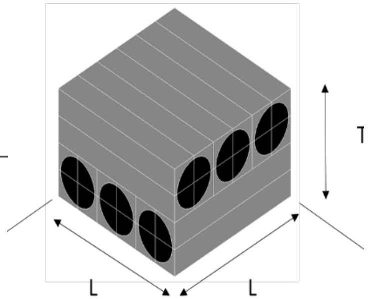


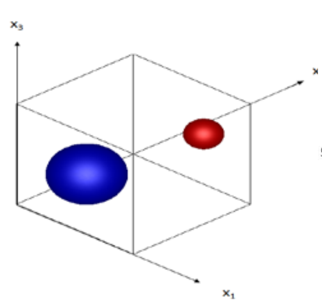
Micromechanics Simulation Challenge

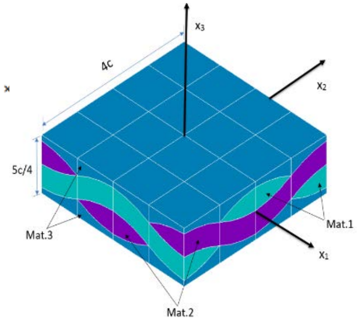
- MAC/GMC, MAC/HFGMC, DIGIMAT, Altair MDS, FVDAM, ESI/VPS, SwiftComp, 3D FEA of RVE with periodic BCs.
- Final report: cdmhub.org/resources/948.
- All data needed for reproducing the results: cdmhub.org/members/project/mmsimulationchalleng/view.
- Level I: accuracy and efficiency of linear thermoelastic properties and local fields.
- SwiftComp achieves versatility and accuracy as 3D FEA with a small fraction of its computing time.

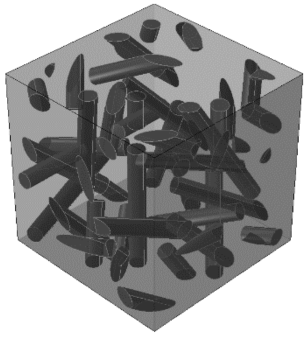




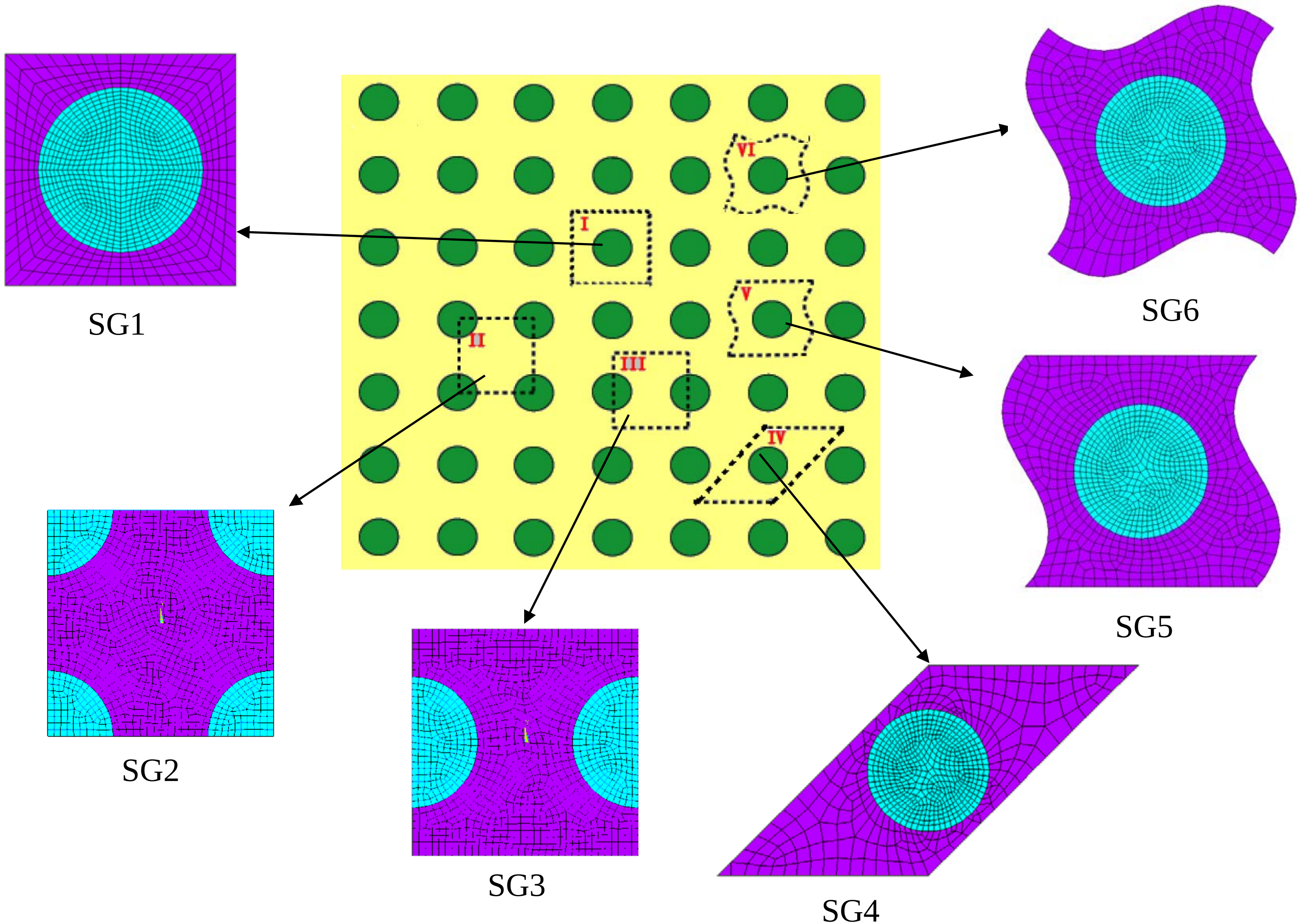




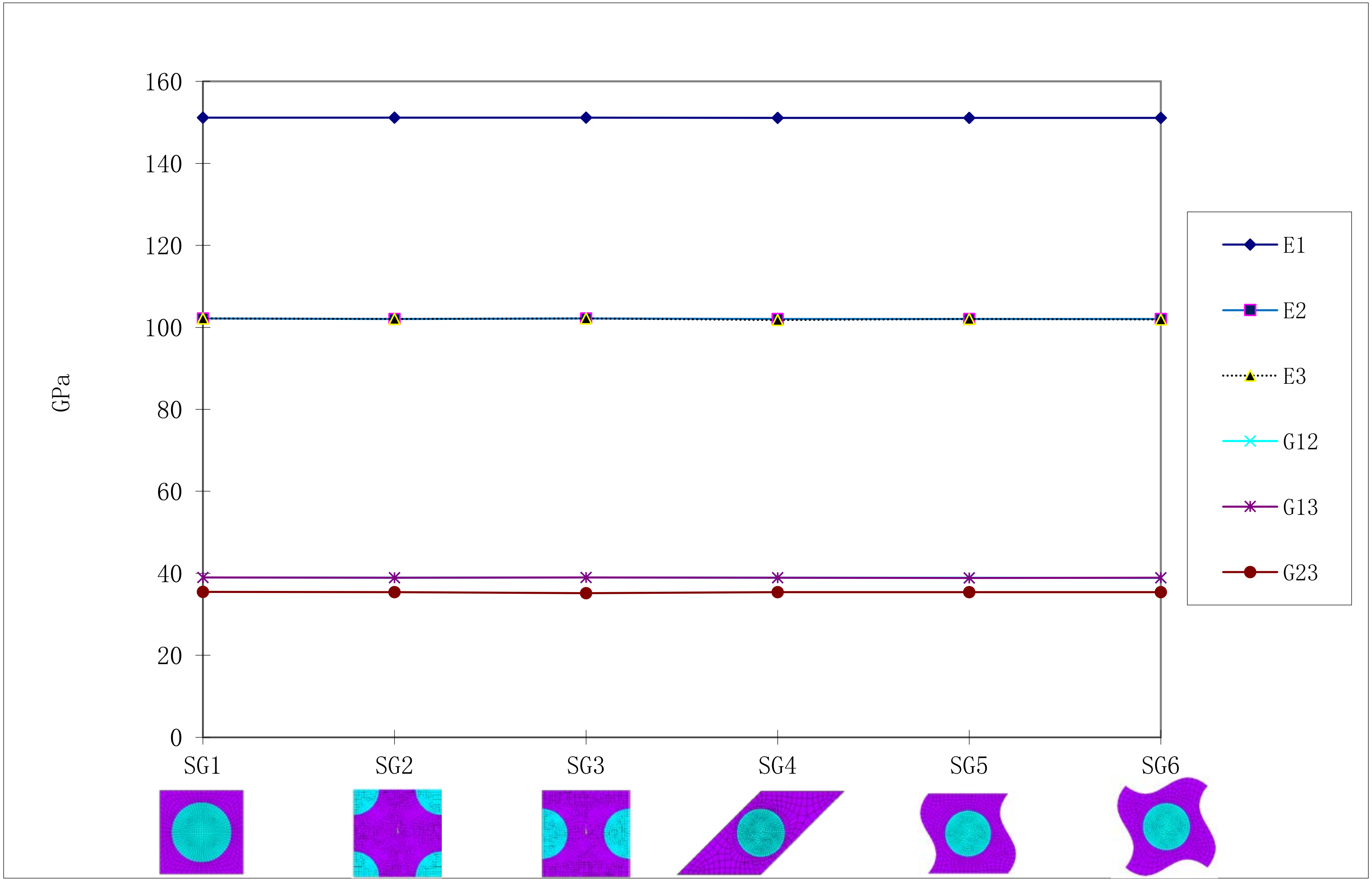




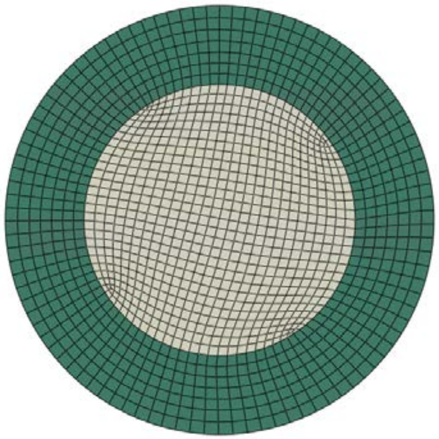
Choice of SG

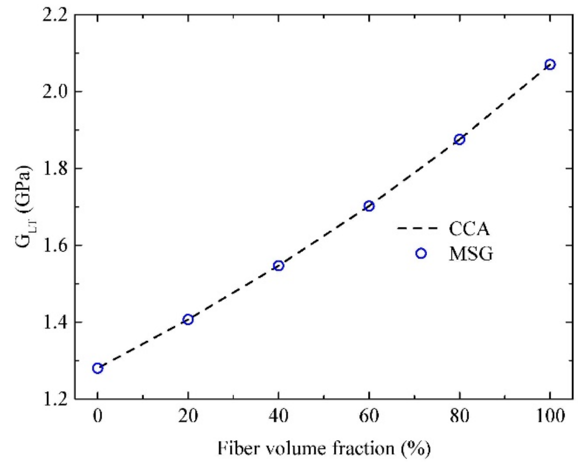


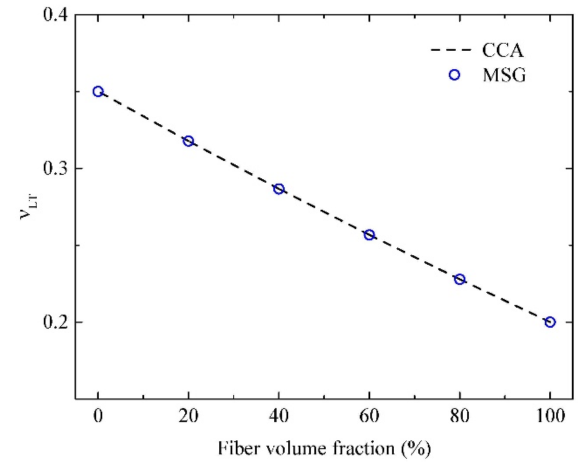
Choice of SG (cont.)

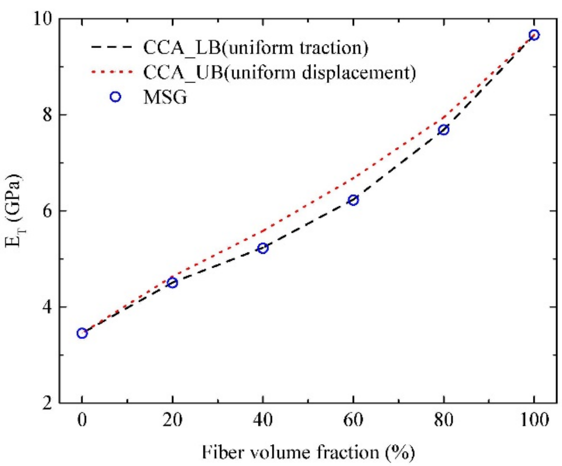


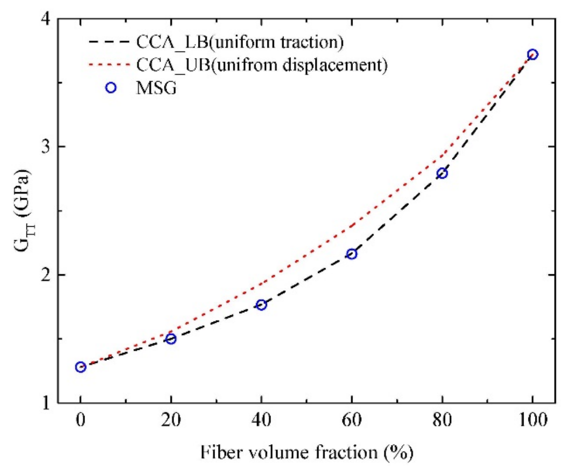
Compare with CCA model

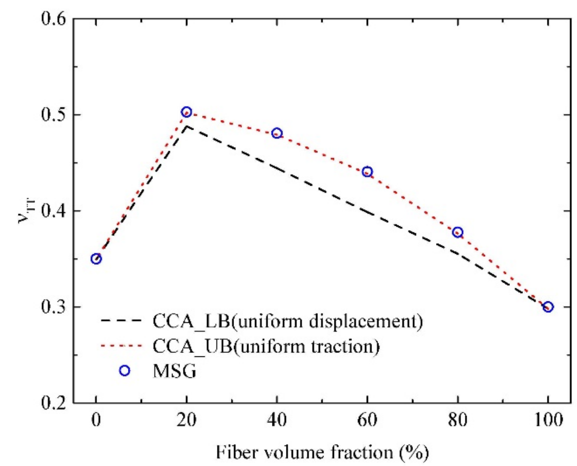




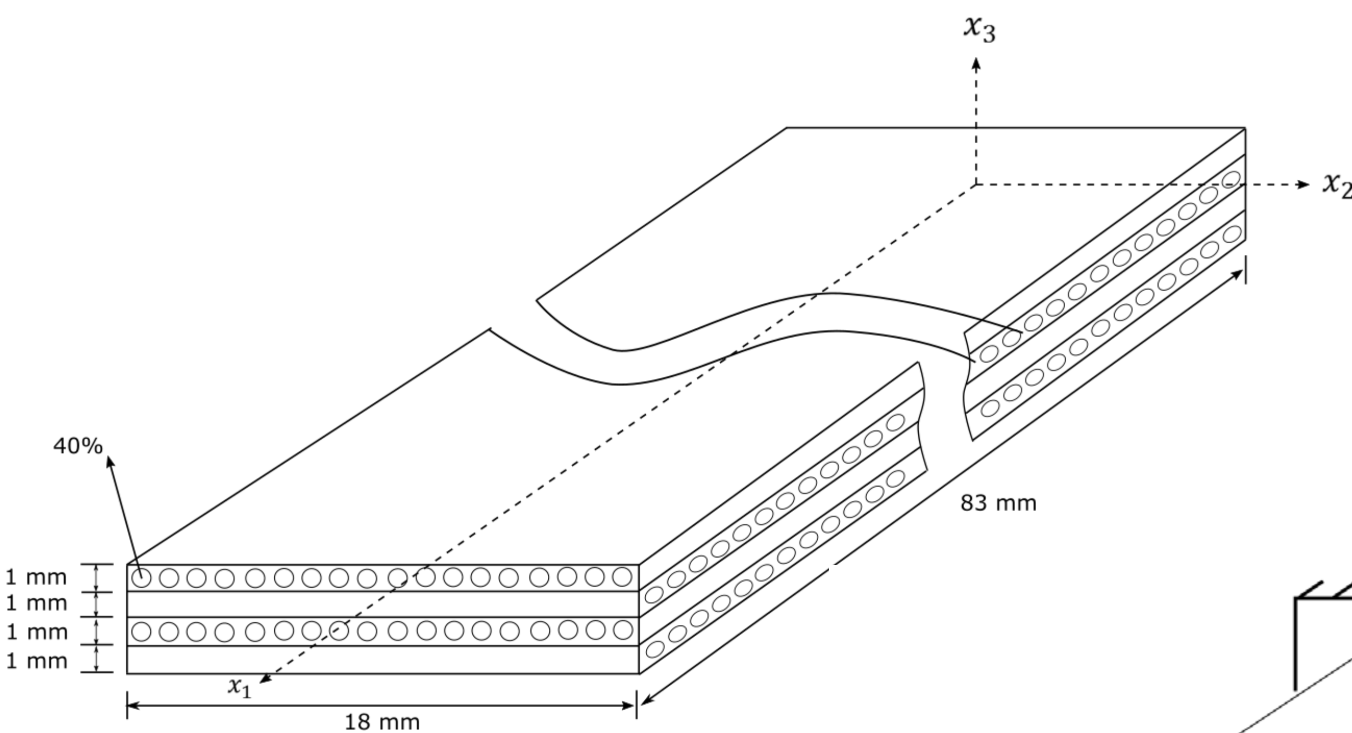


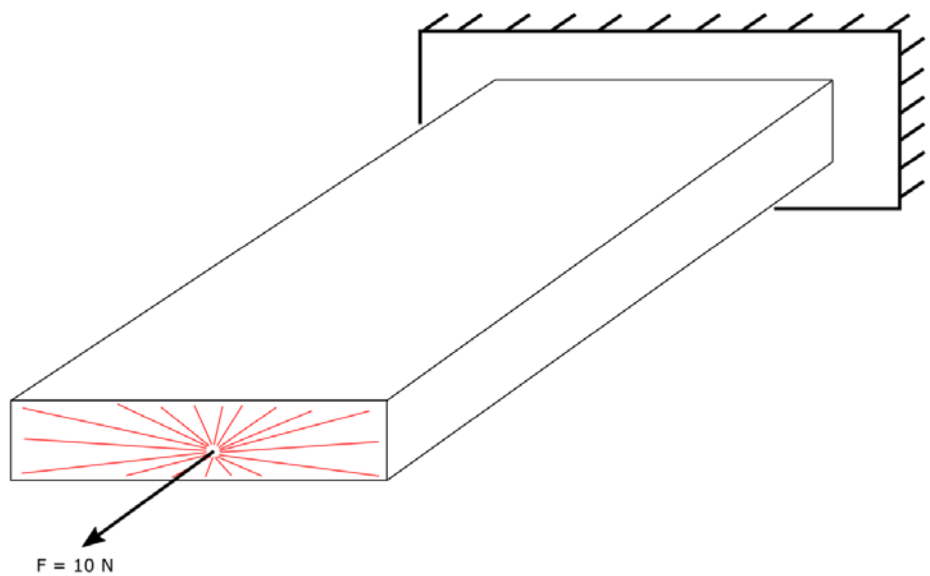






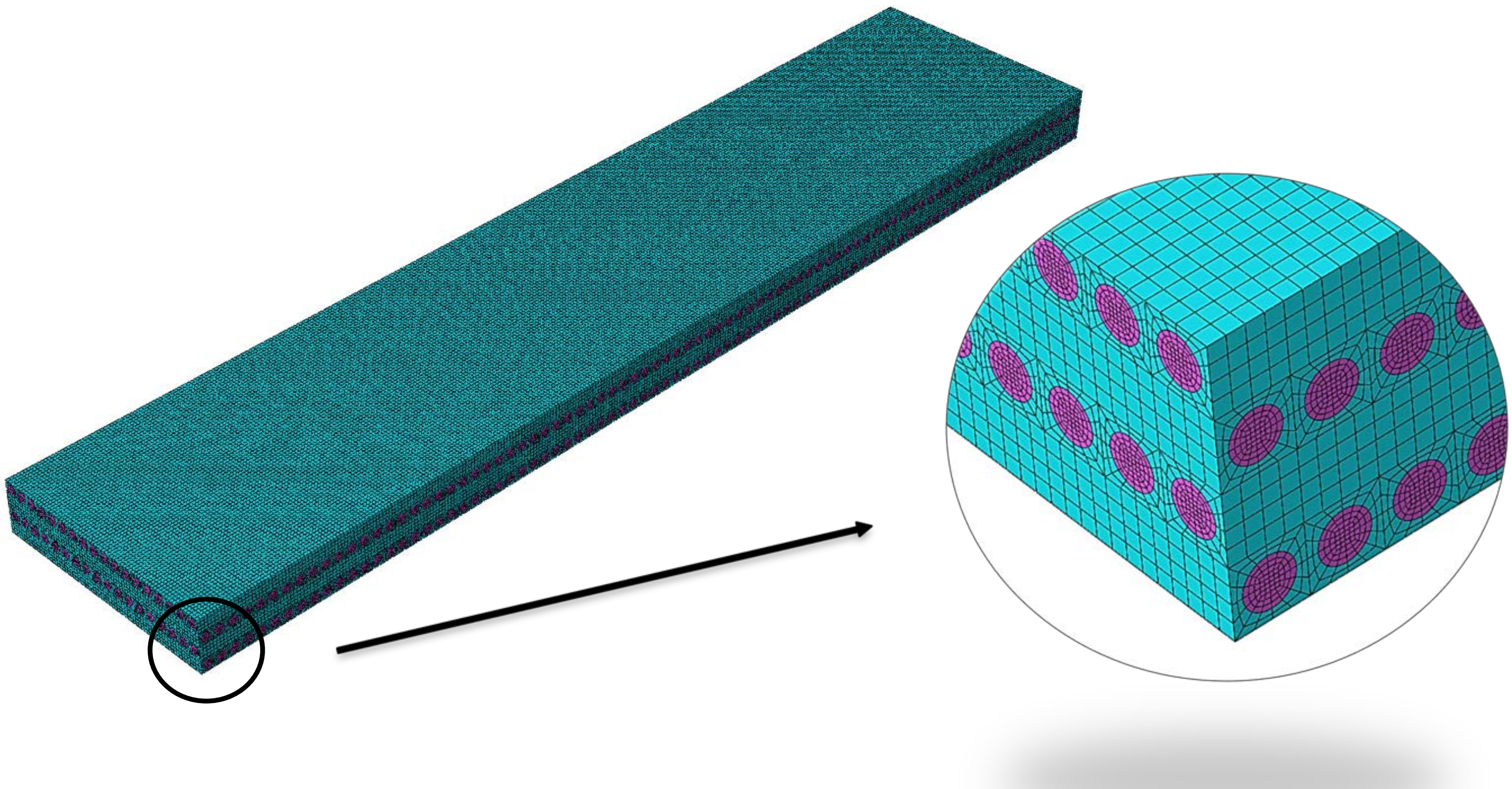
Four Layer Cross-Ply Laminate





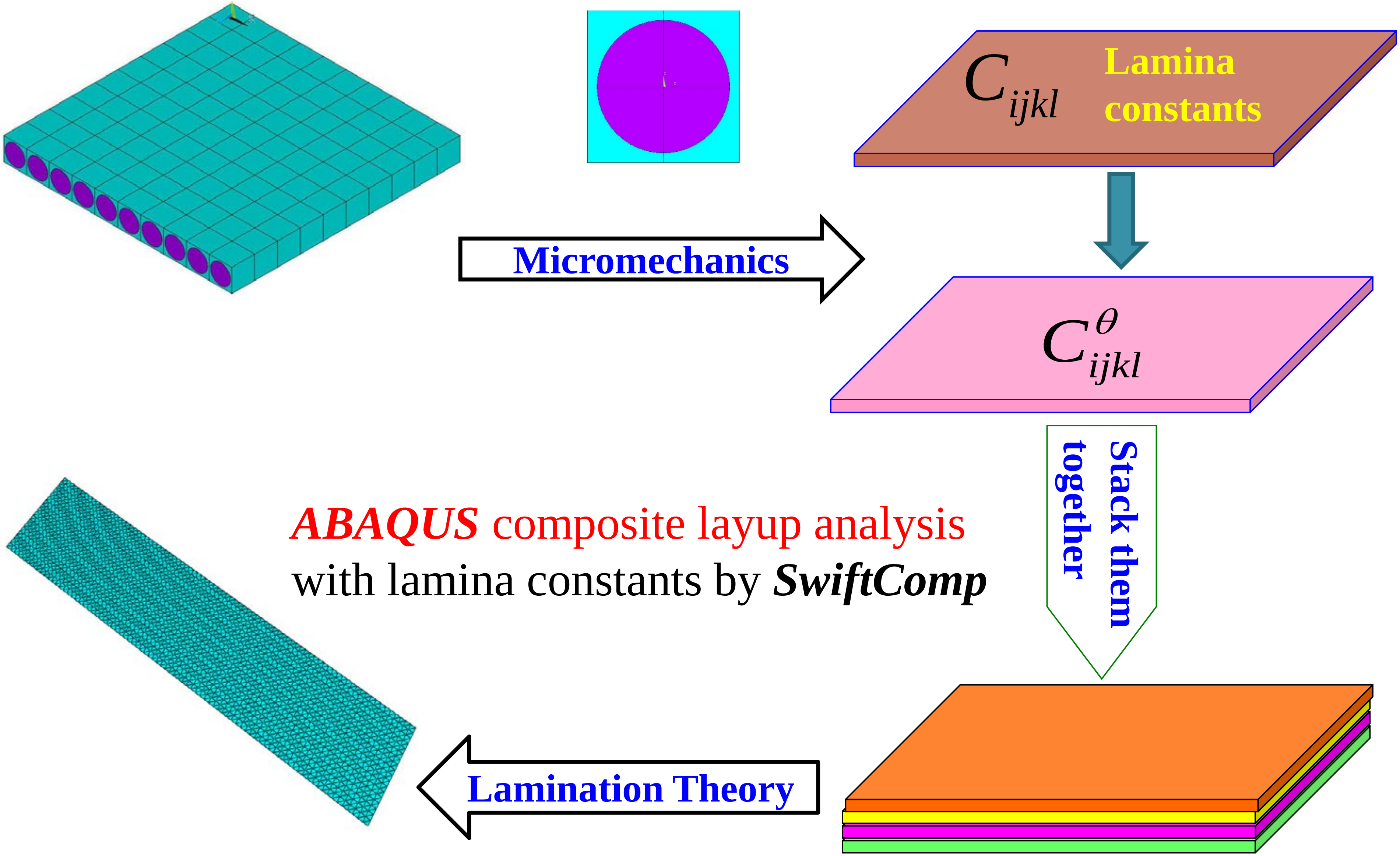
Material	E (GPa)	ν
Fiber	276	0.28
Matrix	4.76	0.37

3D FEA



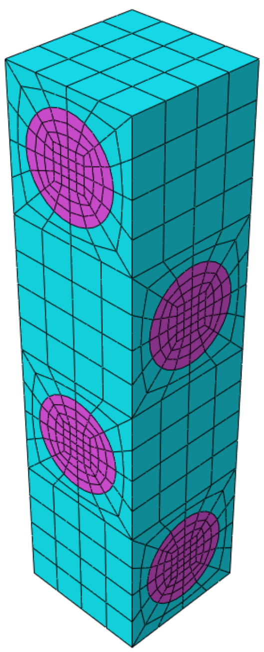
3D FEA	C3D20R	2,294,784	9,319,562

Two-Step Approach



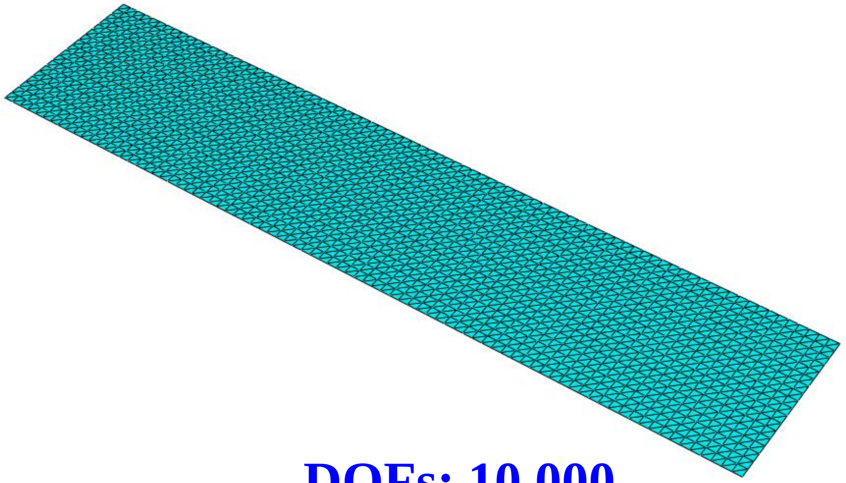
SwiftComp Plate Analysis

Plate SG



DOFs: 23,000

Mesh for plate analysis

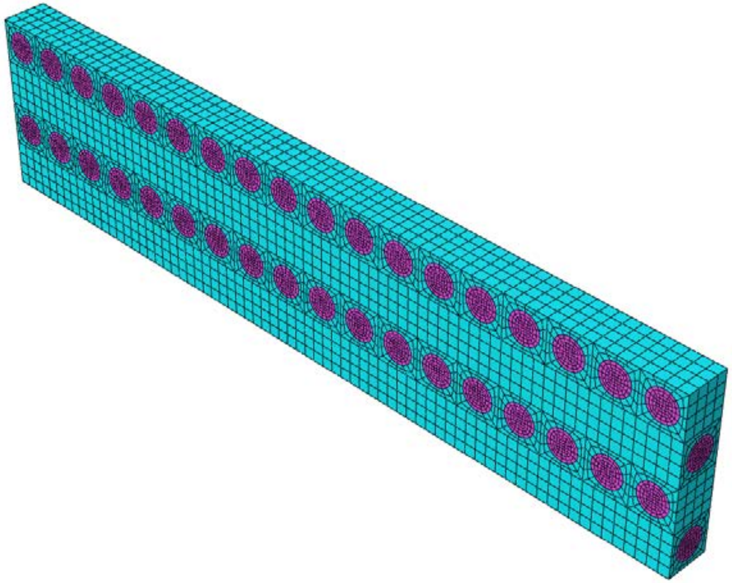


DOFs: 10,000

Same mesh is also used for ABAQUS composite layup analysis

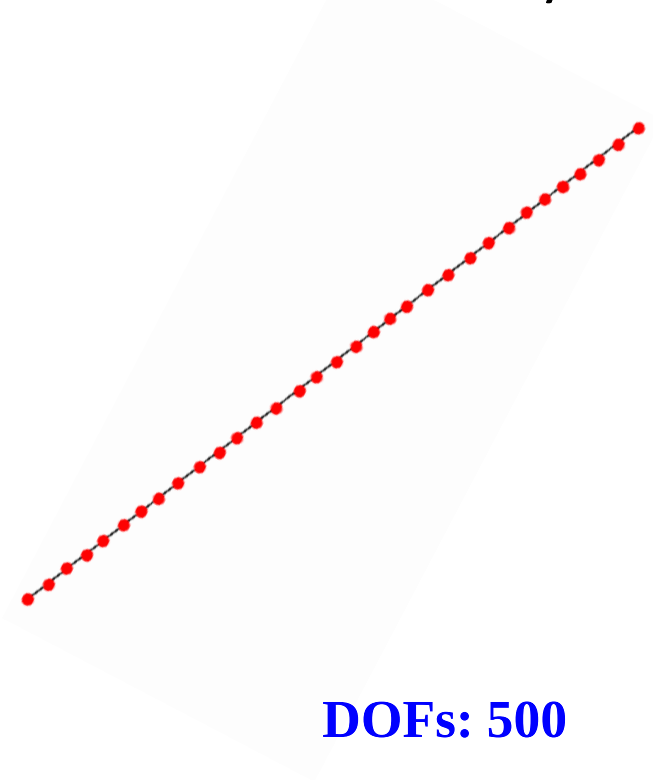
SwiftComp Beam Analysis

Beam SG



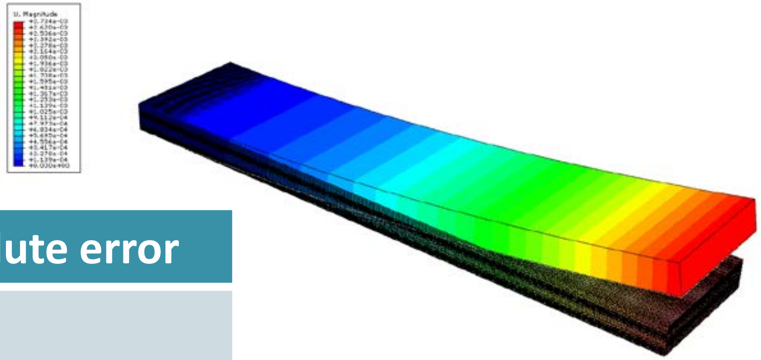
DOFs: 373,000

Mesh for beam analysis



DOFs: 500

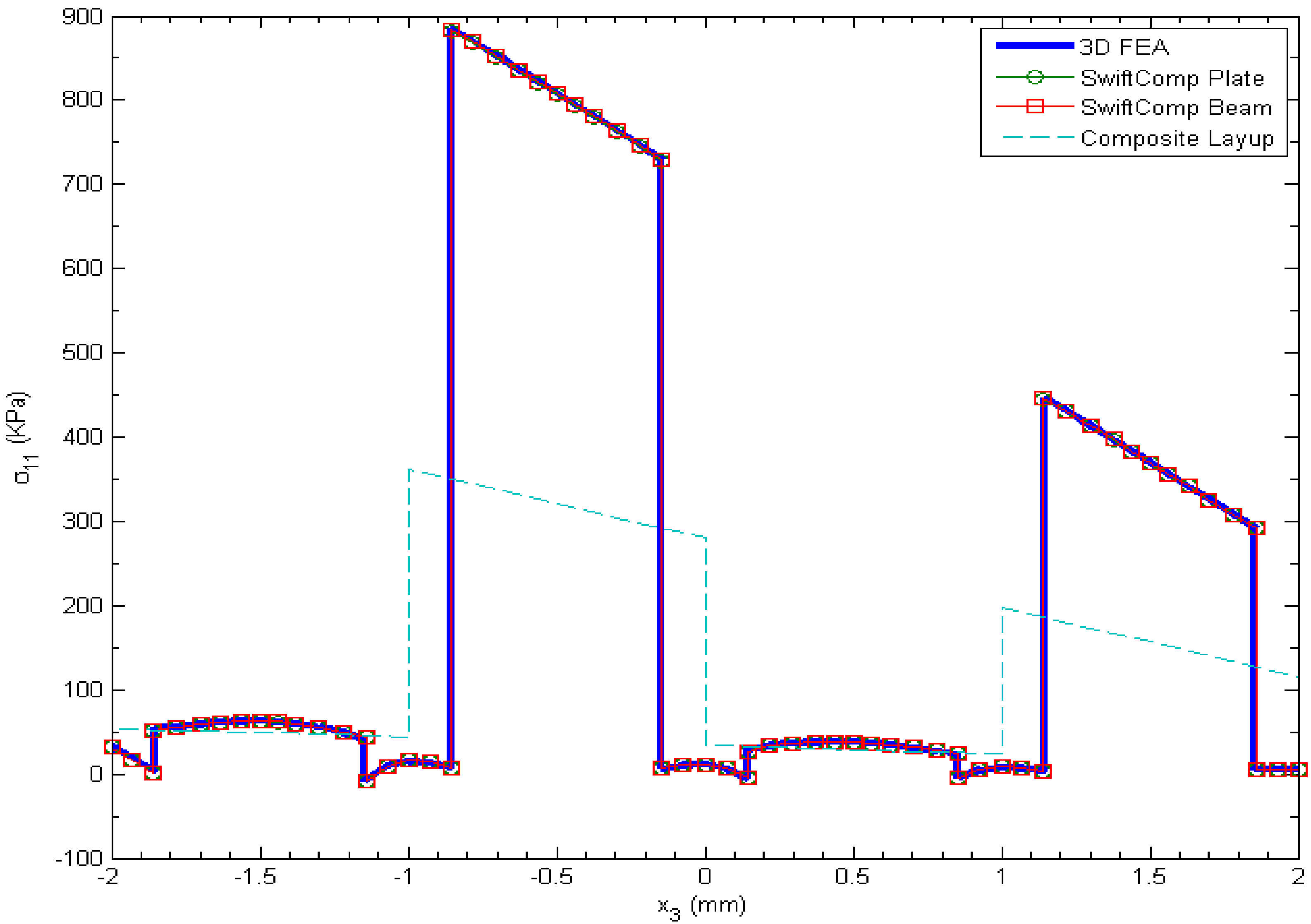
Global Behavior



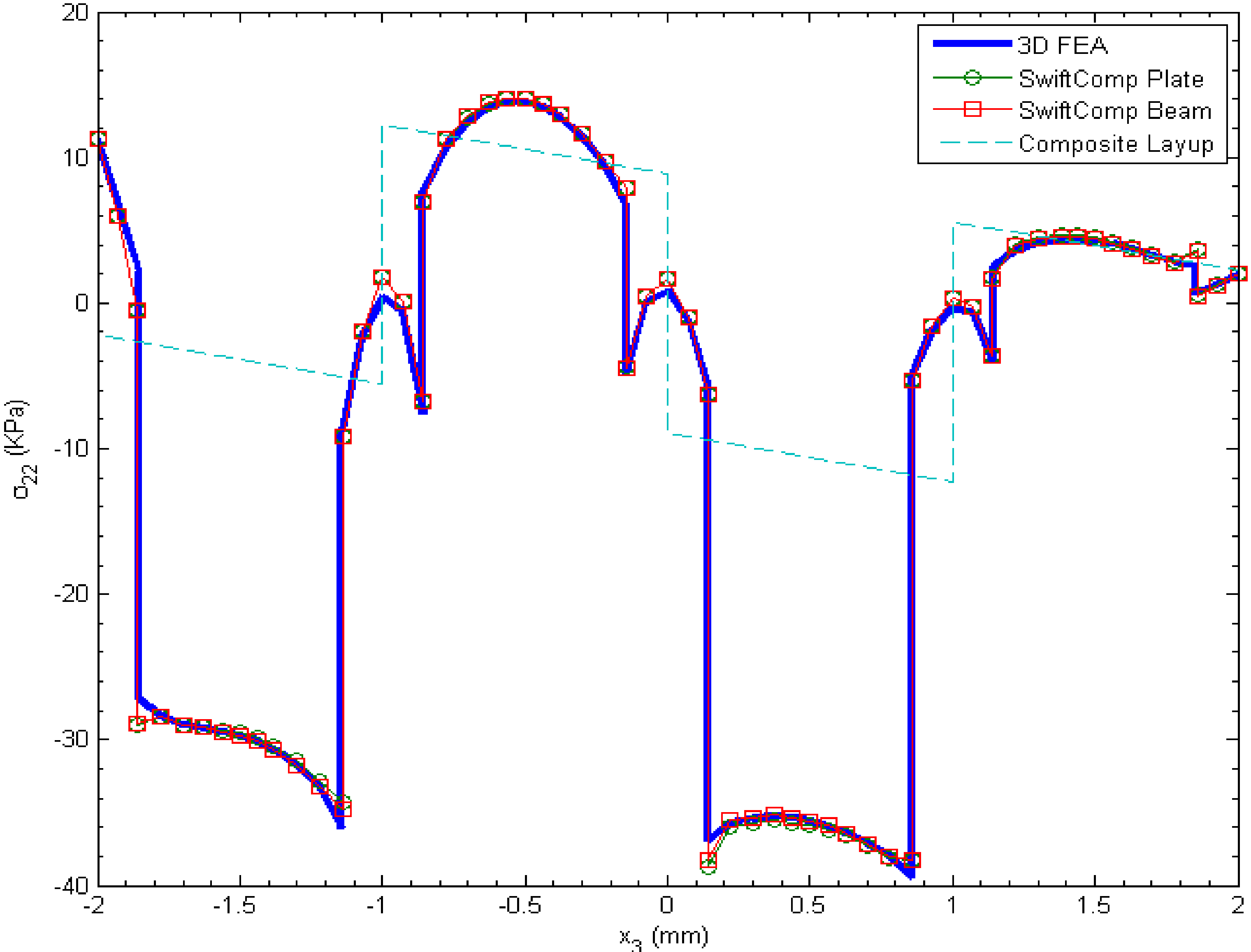
Method	U_1	Absolute error
3D FEA	2.0849×10^{-4}	
SwiftComp Beam	2.0873×10^{-4}	0.1151%
SwiftComp Plate	2.0832×10^{-4}	0.0815%
ABAQUS Composite layup	2.0804×10^{-4}	0.2158%

Method	U_3	Absolute error
3D FEA	2.7124×10^{-3}	
SwiftComp beam	2.7146×10^{-3}	0.0811%
SwiftComp plate	2.7084×10^{-3}	0.1475%
ABAQUS Composite layup	2.5264×10^{-3}	**6.8574%**

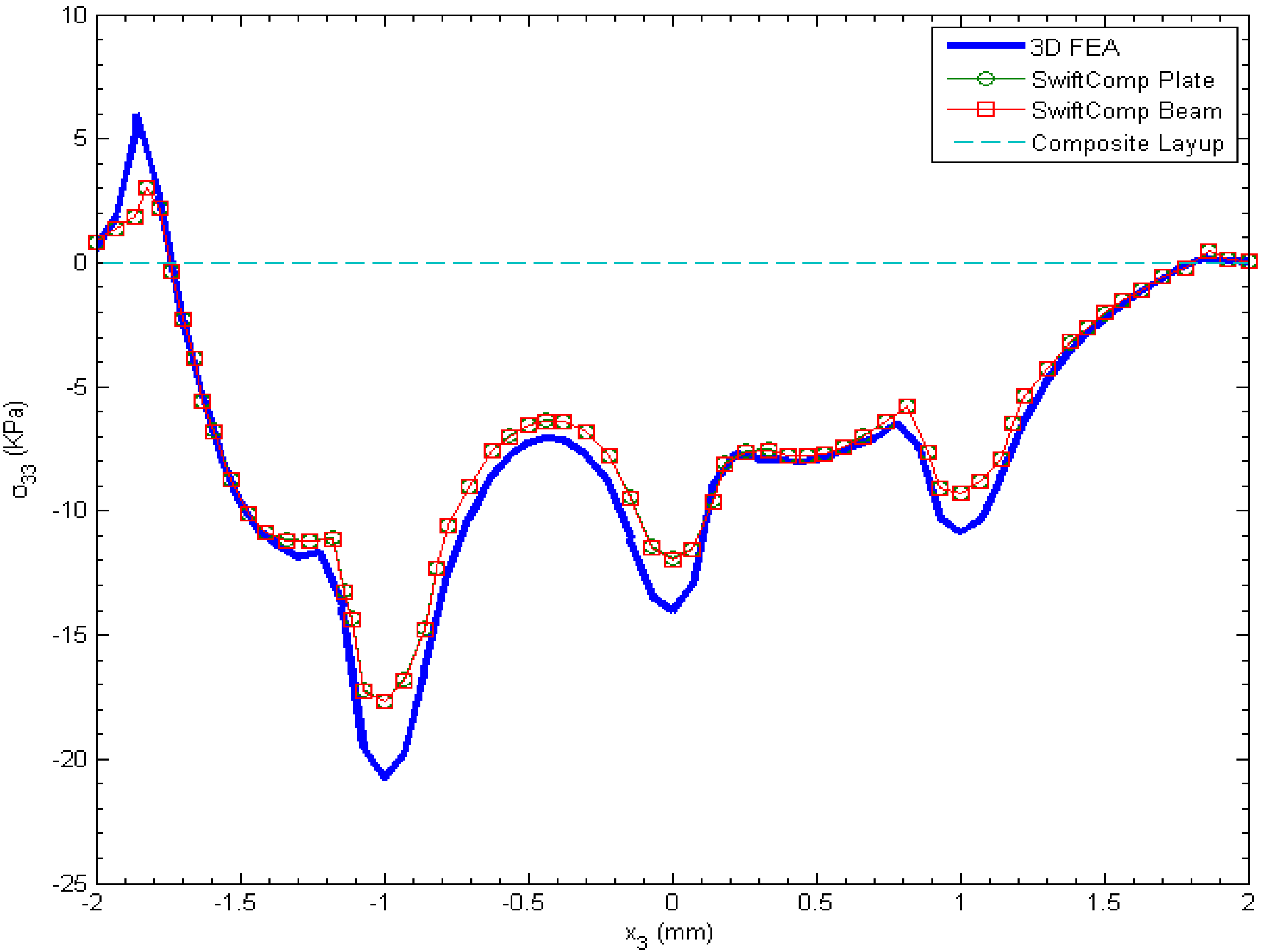
Local Stress Distribution



Local Stress Distribution



Local Stress Distribution



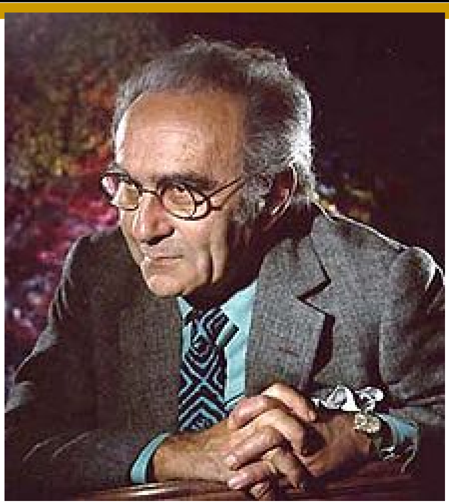
Efficiency

<table>
<tr><th colspan="2">Method</th><th>CPUs</th><th colspan="2">Time</th></tr>
<tr><td colspan="2">3D FEA</td><td>48</td><td colspan="2">7 days 11 hr 37 min</td></tr>
<tr><td colspan="2">ABAQUS Composite Layup</td><td>1</td><td colspan="2">30 sec</td></tr>
<tr><td rowspan="3">SwiftComp Plate Analysis</td><td>Homo.</td><td>1</td><td>6 sec</td><td rowspan="3">40 sec</td></tr>
<tr><td>2D plate analysis</td><td>1</td><td>28 sec</td></tr>
<tr><td>Dehomo.</td><td>1</td><td>6 sec</td></tr>
<tr><td rowspan="3">SwiftComp Beam Analysis</td><td>Homo.</td><td>1</td><td>3 min 14 sec</td><td rowspan="3">4 min 35 sec</td></tr>
<tr><td>1D beam analysis</td><td>1</td><td>0.02 sec</td></tr>
<tr><td>Dehomo.</td><td>1</td><td>1 min 21 sec</td></tr>
</table>

Mechanics of SG & SwiftComp

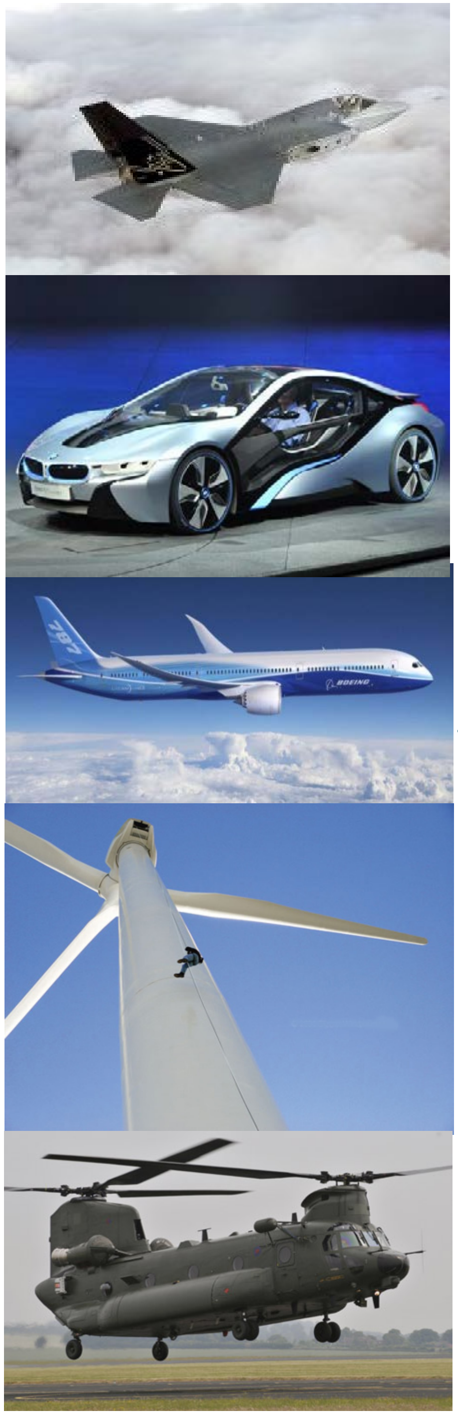
- Directly connects materials genome with structural analysis
- Achieves accuracy of 3D detailed FEA at efficiency of simple engineering models
- Models composites as black aluminum, capturing details as needed and affordable
- Power conventional structural tools with accurate composites modeling

Unifies Structural Mechanics & Micromechanics



Jacob Bronowski
The Creative Aspects of Science

The progress of science is the discovery at each step of a new order which gives unity to what had long seemed unlike. Faraday did this when he closed the link between electricity and magnetism. Clerk Maxwell did it when he linked both with light. Einstein linked time with space, mass with energy, ...
Science is nothing else than the search to discover unity in the wild variety of nature — or more exactly, in the variety of our experience.

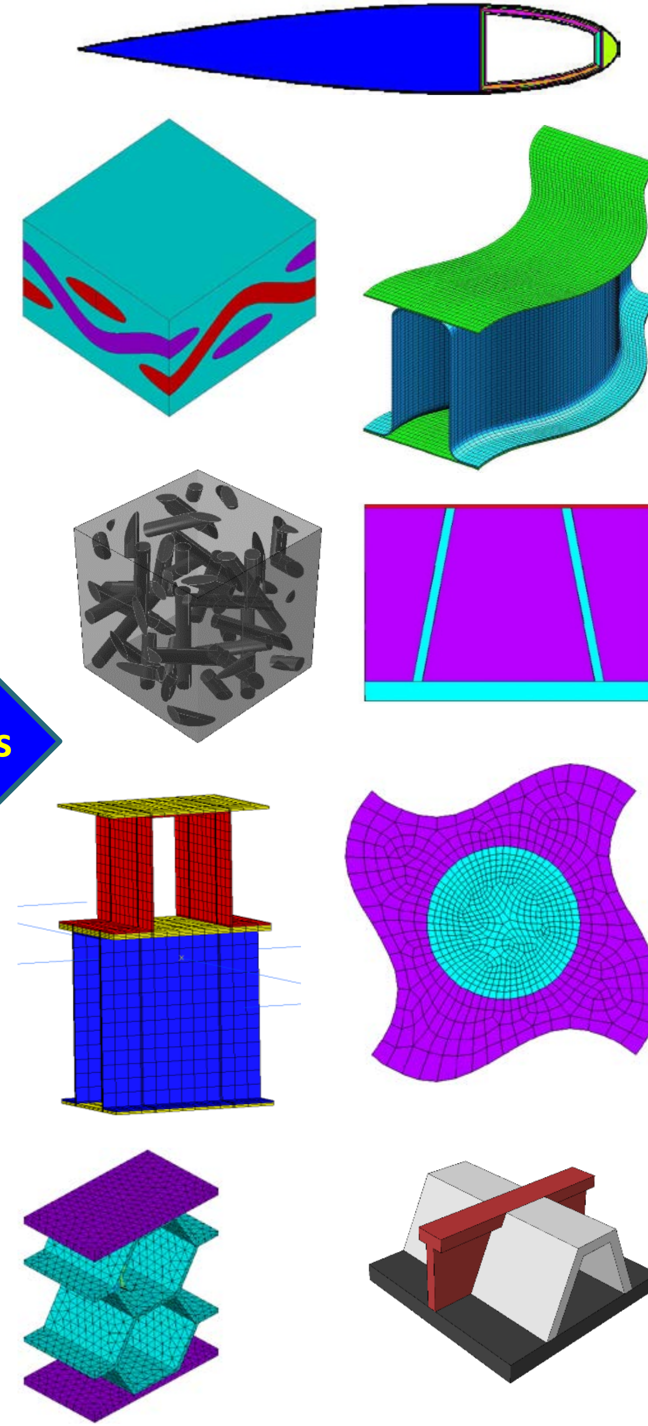


Right Results
Right Away

SwiftComp™
A Purdue Technology

Principle of Minimum Information Loss

- **Virtual testing of materials**
  - Mechanical properties
  - Multifunctional properties
- **Multiscale modeling of structures**
  - Composite structures
  - Stiffened structures
  - Build-up structures



Structure Genome vs RVE

- Structure Genome (SG): smallest mathematical building block; structural properties (beams/plates/shells/3D bodies)
- RVE: volume entirely typical of material on average and contains a sufficient number of inclusions for effect properties independent of BCs; 3D RVE required for 3D properties
- Equivalent: 3D bodies featuring 3D periodicity

MSG, Asymptotic Homogenization (AH), & RVE Analysis

- RVE analysis with periodic BCs is the same as AH, same as MSG for 3D RVEs (in theory)
- MSG: no BCs in terms of displacements/tractions, automatically satisfying Hill-Mandel condition
- No assumptions for SG and local fields
- MSG unifies structural modeling and micromechanics
- MSG can handle partially periodic or aperiodic structures
- MSG can directly construct models for beams/plates/shells