Generalized Free Edge Stress Analysis Using Mechanics of Structure Genome

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The Institute for iacmi Advanced Composites Manufacturing Innovation

PURDUE AERONAUTICS & ASTRONAUTICS

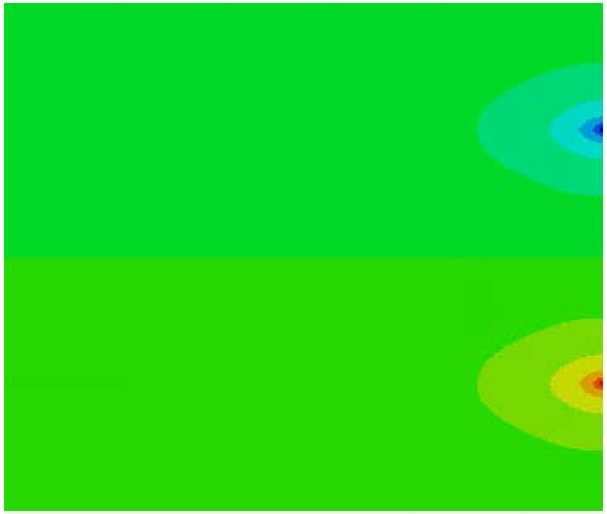
Multiscale StructuralMechanics

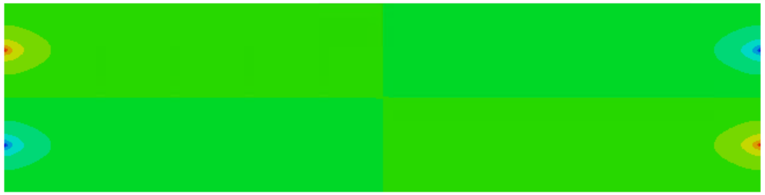
COMPOSITES DESIGN & MANUFACTURING HUB

Free Edge effect

Classical Laminated Plate Theory:

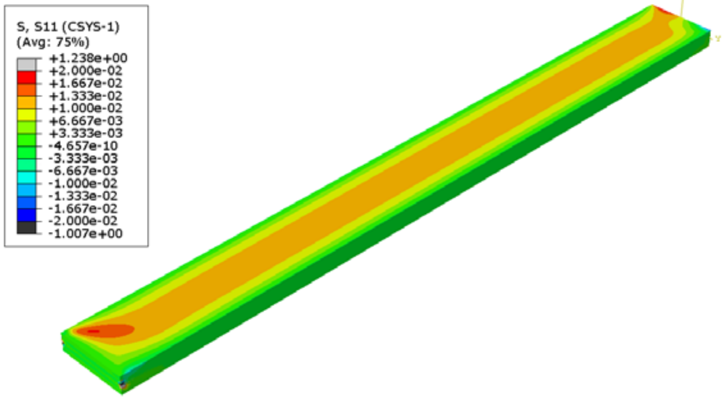
- Assume homogeneous layer properties and plane stress state.
- Material properties will drastically change at the layer interfaces.
- Due to Poisson's ratio of different layers, localized stress singularity at the free edges.





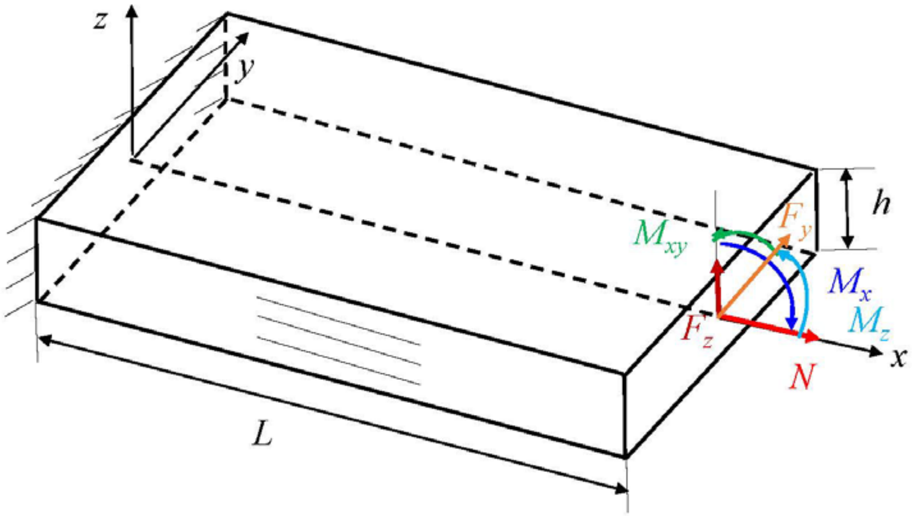
σ_{xz} in an angle-ply laminate $(\pm 45°)$s under uniaxial extension [1]

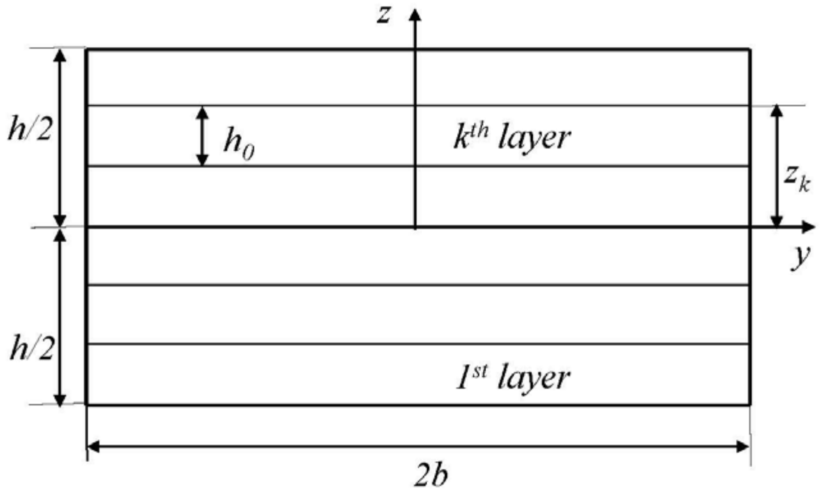
Review

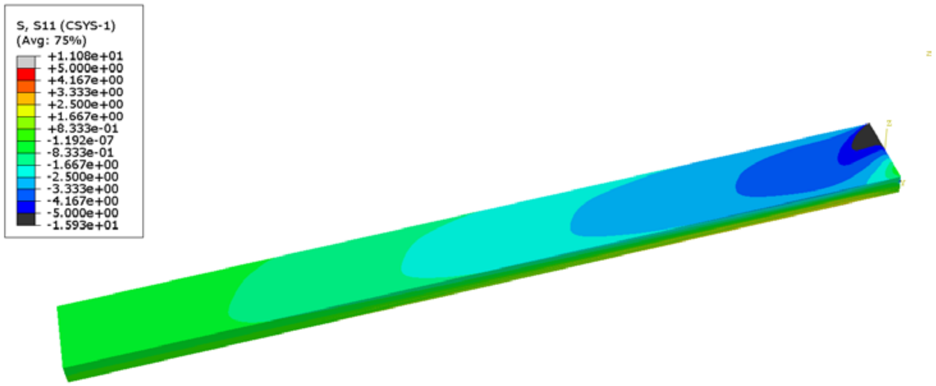


- Quasi-3D models
  - Zero gradient along the x direction
  - Equivalent Single Layer(ESL), Layer-Wise(LW), displacement-based, stress-based
- 2D plate models
  - ESL, LW, displacement-based, stress-based
  - In Carrera’s Unified formulation, 4^{th}-order LW models should be used
- 3D numerical methods
  - Generation of new and efficient meshing approaches
  - Developing special purpose element for dealing with the singular stress field

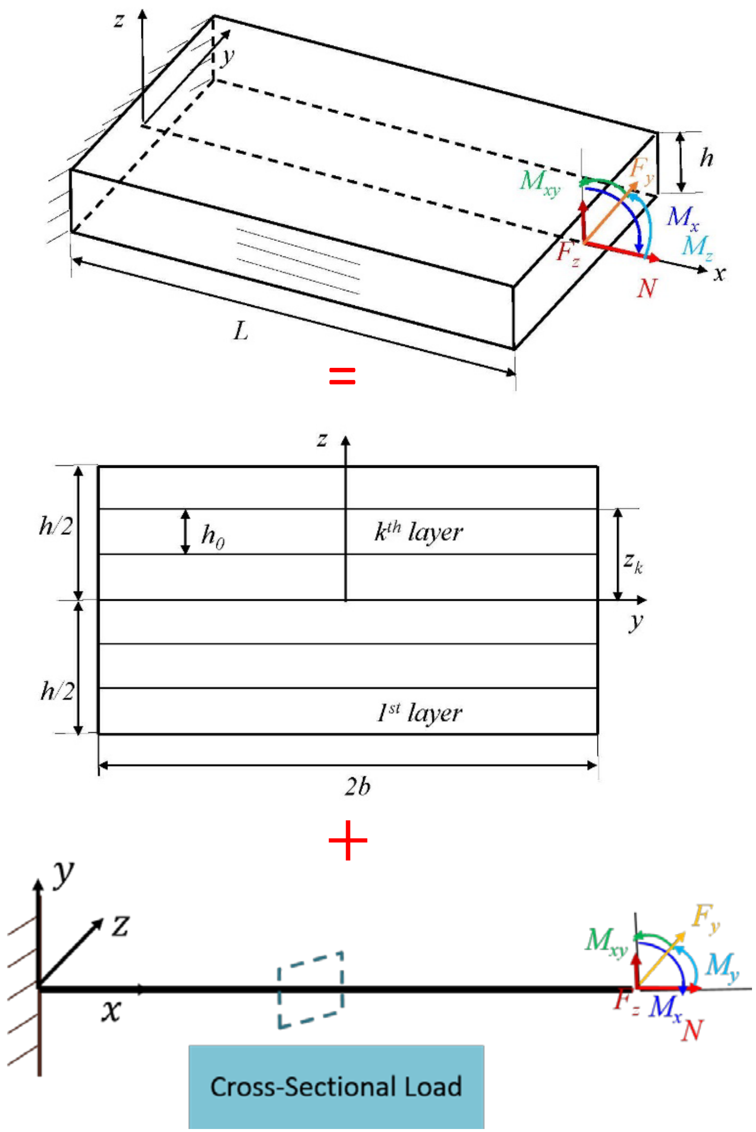
Generalized Free Edge Problem

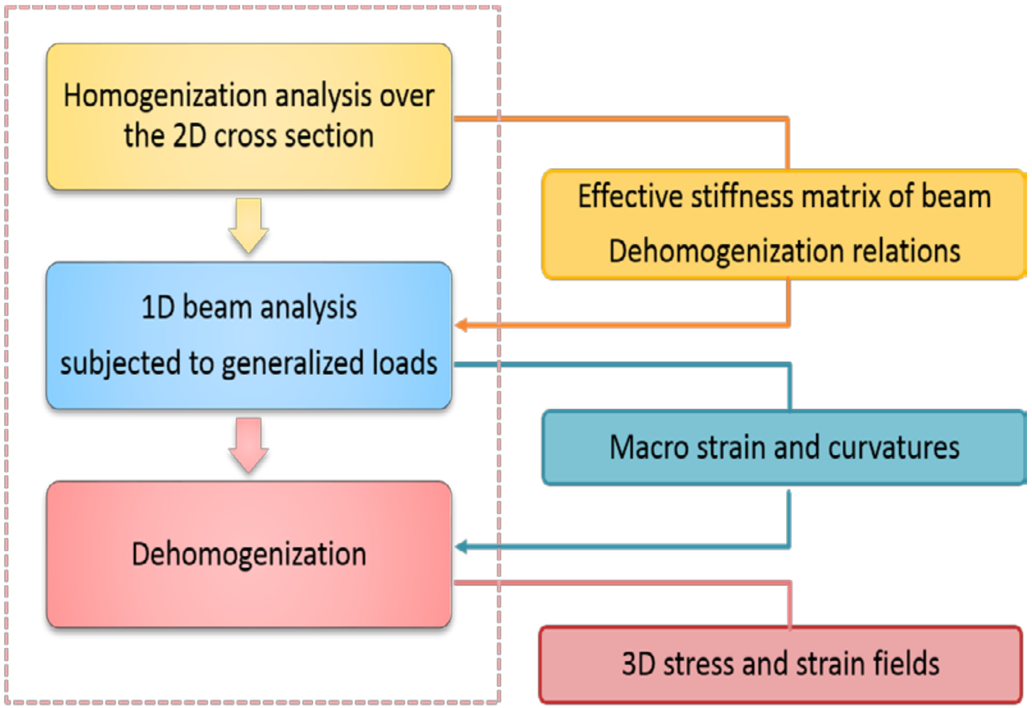






Mechanics of Structure Genome





MSG-Based Free-Edge Stress Analysis

- 3D displacements:

$$u(x,y,z) = \bar{u}(x) - z\bar{w}_{,x}(x) - y\bar{v}_{,x}(x) + U(x,y,z)$$
$$v(x,y,z) = \bar{v}(x) - z\bar{\phi}(x) + V(x,y,z)$$
$$w(x,y,z) = \bar{w}(x) + y\bar{\phi}(x) + W(x,y,z)$$
$$\bar{\phi} = \langle w_{,y} - v_{,z} \rangle$$

- 3D strains:

$$\varepsilon_x(x,y,z) = \varepsilon + z\kappa_2 - y\kappa_3 + U_{,x}(x,y,z)$$
$$\varepsilon_y(x,y,z) = V_{,y}(x,y,z)$$
$$\varepsilon_z(x,y,z) = W_{,z}(x,y,z)$$
$$\gamma_{xy}(x,y,z) = -z\kappa_1 + U_{,y}(x,y,z) + V_{,x}(x,y,z)$$
$$\gamma_{xz}(x,y,z) = y\kappa_1 + U_{,z}(x,y,z) + W_{,x}(x,y,z)$$
$$\gamma_{yz}(x,y,z) = V_{,z}(x,y,z) + W_{,y}(x,y,z)$$

$$\varepsilon = \bar{u}_{,x}(x),\ \kappa_1 = \bar{\phi}_{,x}(x),\ \kappa_2 = -\bar{w}_{,xx}(x),\ \text{and}\ \kappa_3 = \bar{v}_{,xx}(x)$$

MSG-Based
Free-Edge Stress Analysis

- Variational statement:

$$\delta\Pi = \overline{\delta W}$$

$$\Pi = \frac{1}{2}\int_0^L \langle \varepsilon^{\mathrm{T}} D \varepsilon \rangle \mathrm{d}x$$

- Warping functions are governed by
  Variational statement of the cross-sectional analysis:

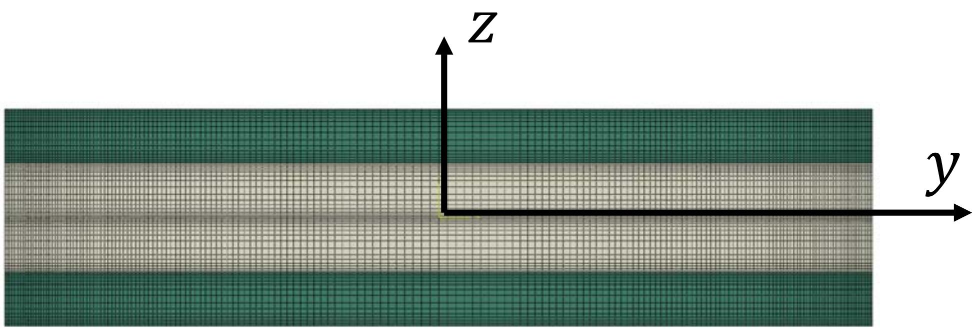
$$\delta E = \delta\left(\frac{1}{2}\langle \varepsilon^{\mathrm{T}} D \varepsilon \rangle\right) = \frac{1}{2}\delta\langle \varepsilon^{\mathrm{T}} \sigma \rangle = 0$$

$$\varepsilon = \begin{bmatrix} \varepsilon_x & \varepsilon_y & \varepsilon_z & \gamma_{yz} & \gamma_{xz} & \gamma_{xy} \end{bmatrix}^{\mathrm{T}}$$

$$\sigma = \begin{bmatrix} \sigma_x & \sigma_y & \sigma_z & \tau_{yz} & \tau_{xz} & \tau_{xy} \end{bmatrix}^{\mathrm{T}}$$

Extension of $[45/-45]_s$ Laminates

➢ Cross Section

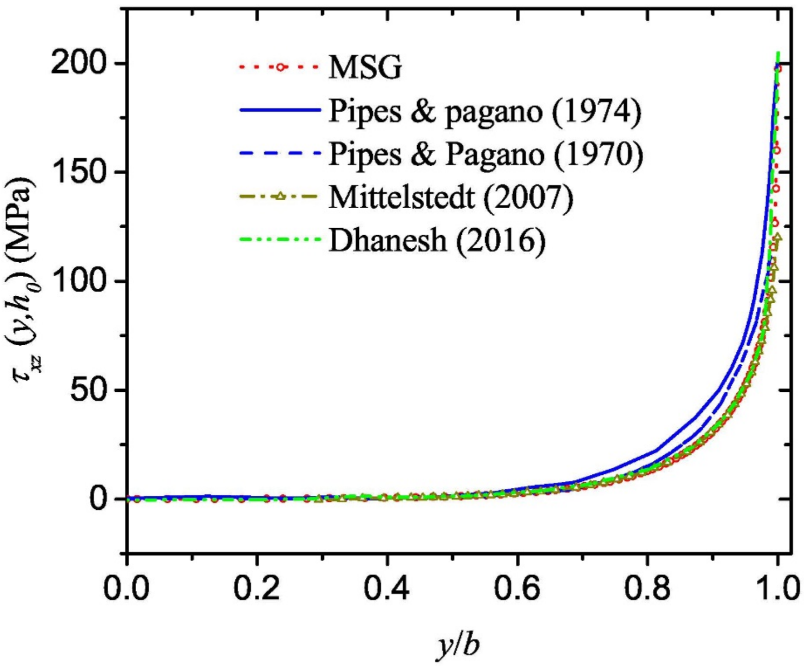


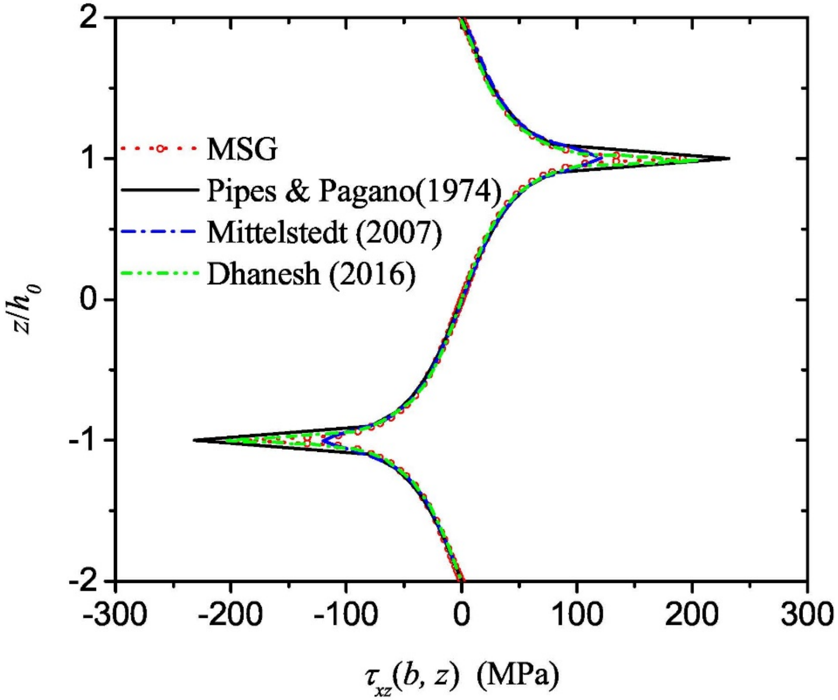
Width=8 h_0;
h_0=0.5 mm

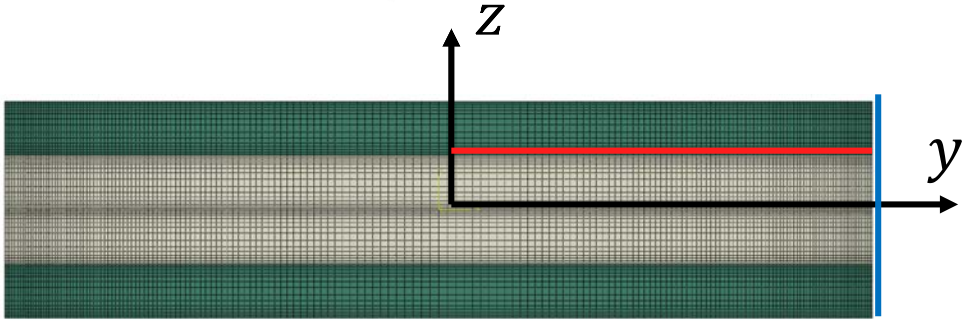
➢ Apply macro extension strain (0.01)

E_1(MPa)	E_2(MPa)	E_3(MPa)	G_{12}(MPa)	G_{13}(MPa)	G_{23}(MPa)	v_{12}	v_{13}	v_{23}
137895.1	14478.99	14478.99	5860.544	5860.544	5860.544	0.21	0.21	0.21

Extension of $[45/-45]_s$ Laminates





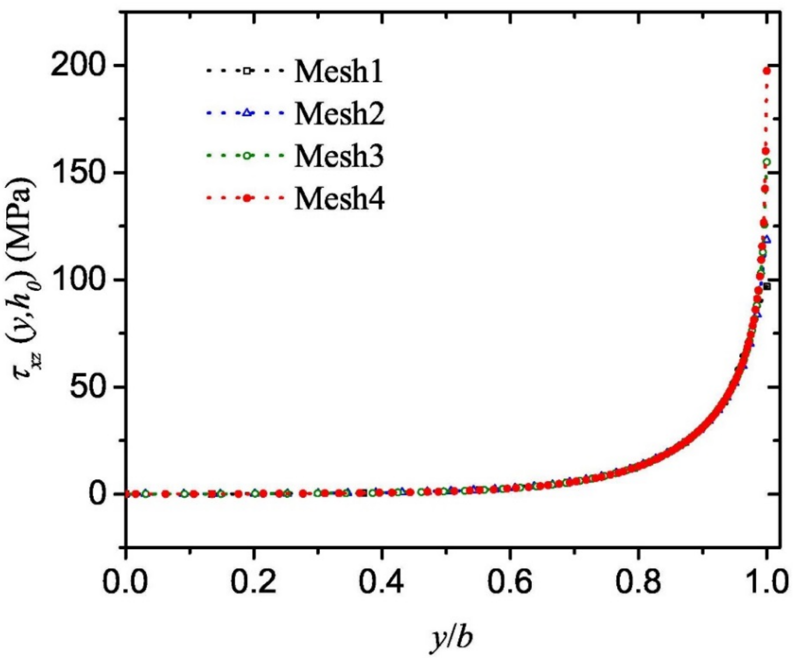


Path (y,h_0)

Path (b,z)

Extension of [45/-45]$_s$ Laminates

- Convergence



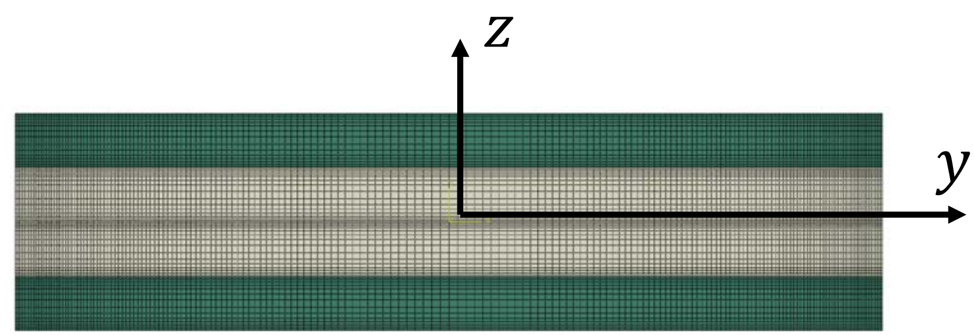
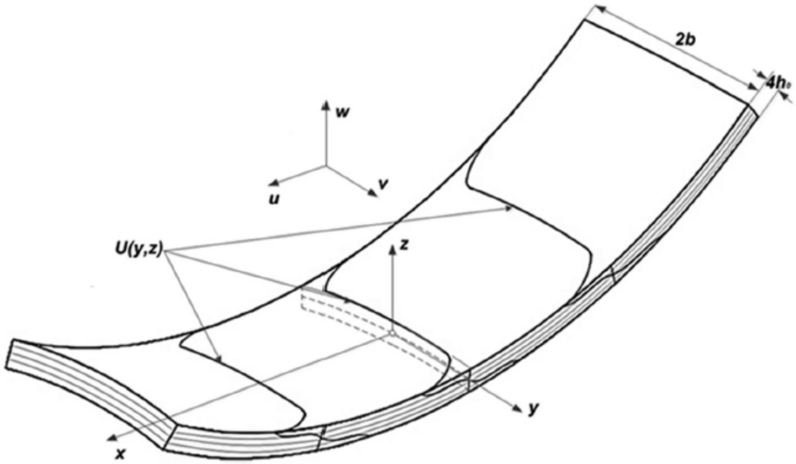


Table 1. Computation time using different mesh configurations

Mesh	Mesh1	Mesh2	Mesh3	Mesh4
b_{min}	0.16	0.08	0.02	0.02
h_{min}	0.08	0.08	0.03	0.03
Node number	735	2059	4387	13013
Element number	680	1960	4240	4240
Computation time(s)	0.93	2.43	5.53	12.90

Anticlastic Bending of $[45/-45]_s$ Laminates

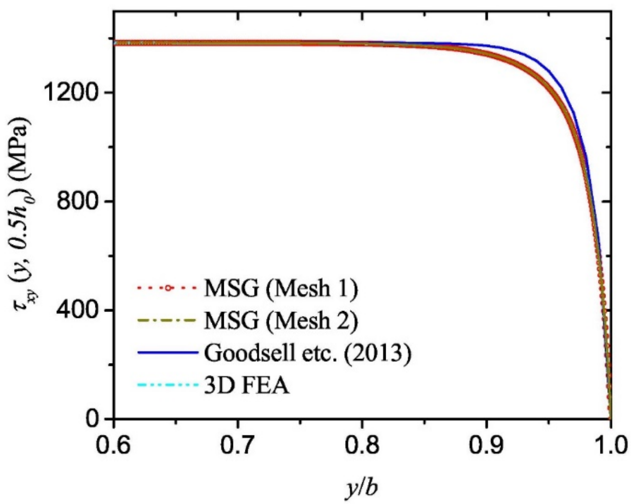


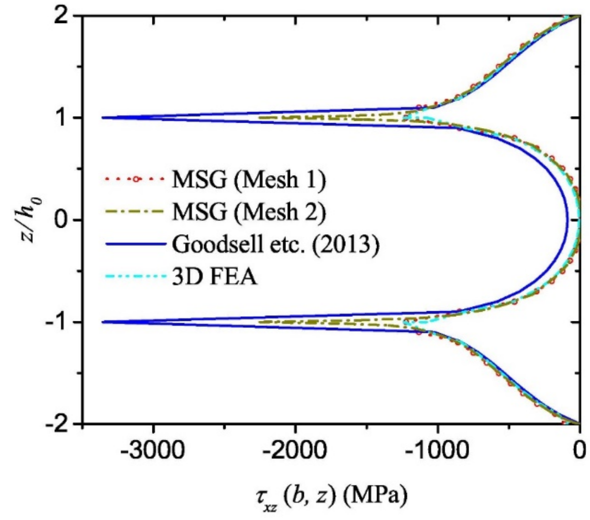
Width=100 h_0;
h_0=0.127 mm

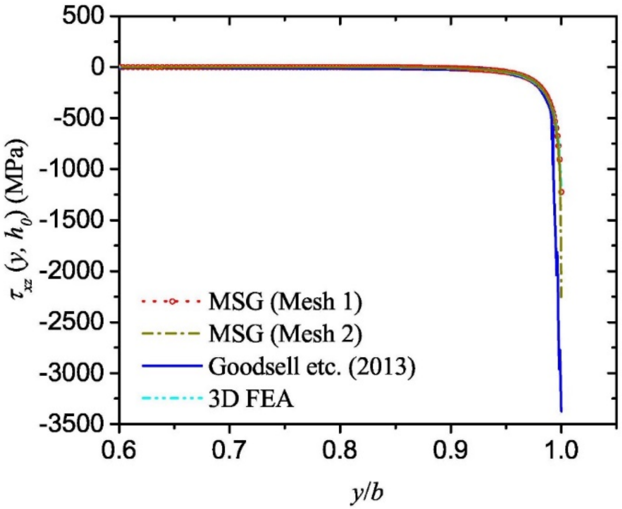
- Apply unit macro beam curvature k_2

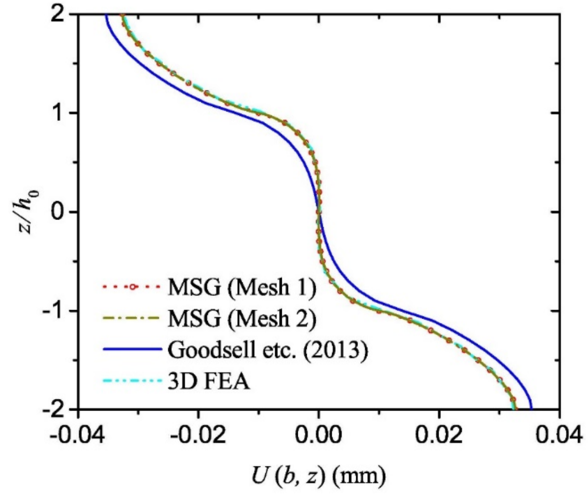
E_1(MPa)	E_2(MPa)	E_3(MPa)	G_{12}(MPa)	G_{13}(MPa)	G_{23}(MPa)	v_{12}	v_{13}	v_{23}
174600	7927	7927	4482	4482	2758	0.34	0.34	0.45

Anticlastic Bending of $[45/-45]_s$ Laminates



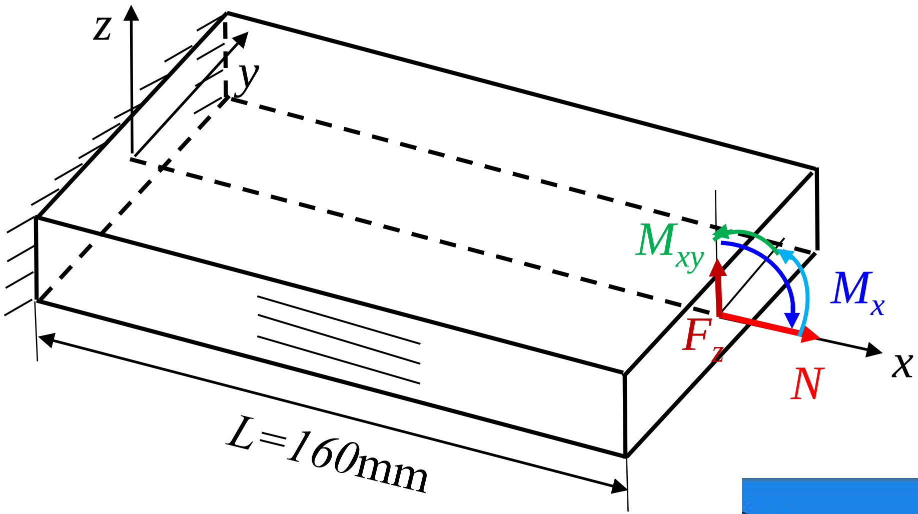


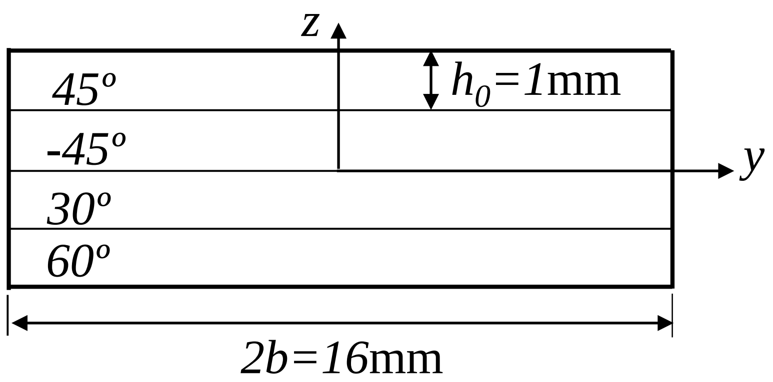


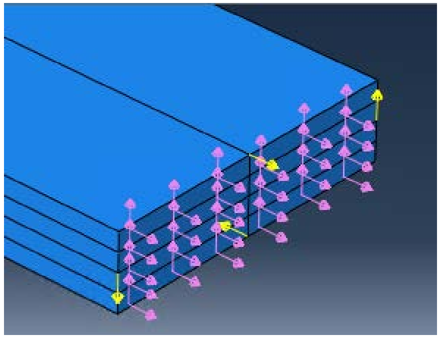


Combined Load on [60/30/-45/45]

- Boundary Conditions $N = 100N, F_z = 1N,\ M_{xy} = M_x = 0.1N{\cdot}\mathrm{m}$



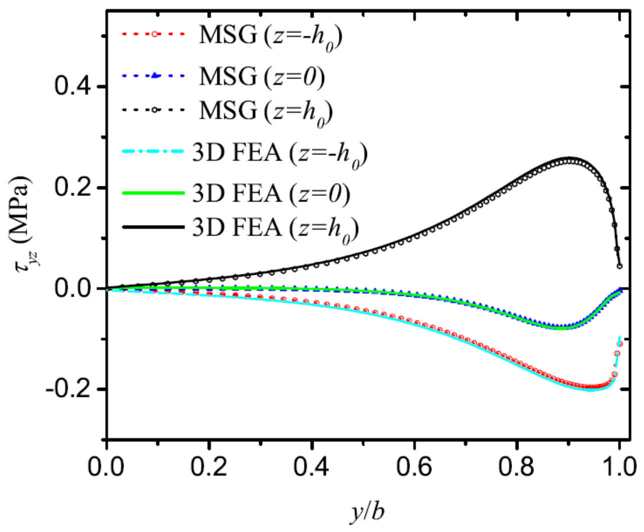


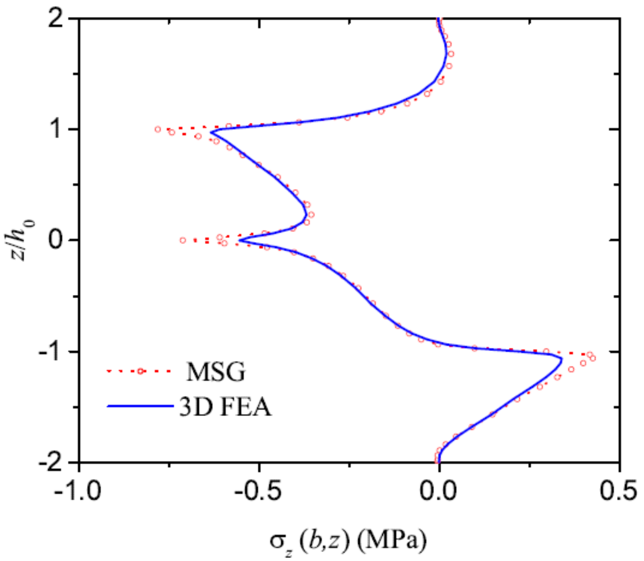


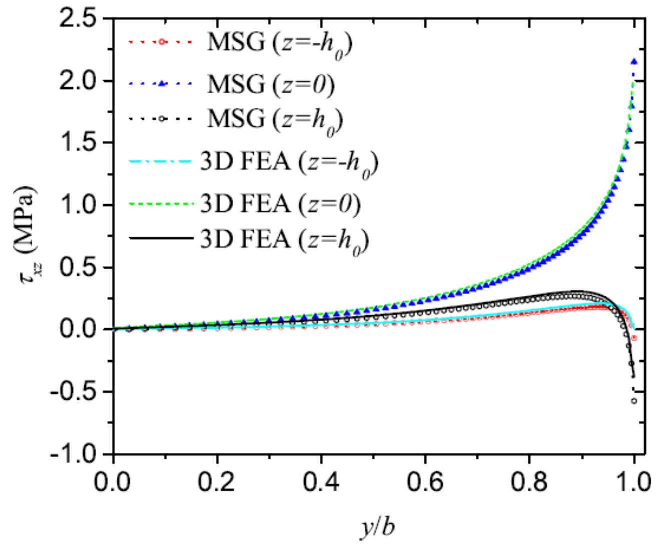
- Materials

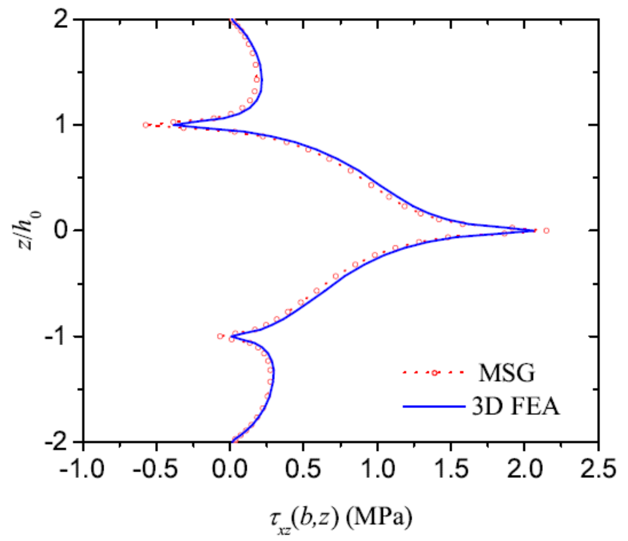
E_1(MPa)	E_2(MPa)	E_3(MPa)	G_{12}(MPa)	G_{13}(MPa)	G_{23}(MPa)	v_{12}	v_{13}	v_{23}
137895.1	14478.99	14478.99	5860.544	5860.544	5860.544	0.21	0.21	0.21

Combined Load on [60/30/-45/45]



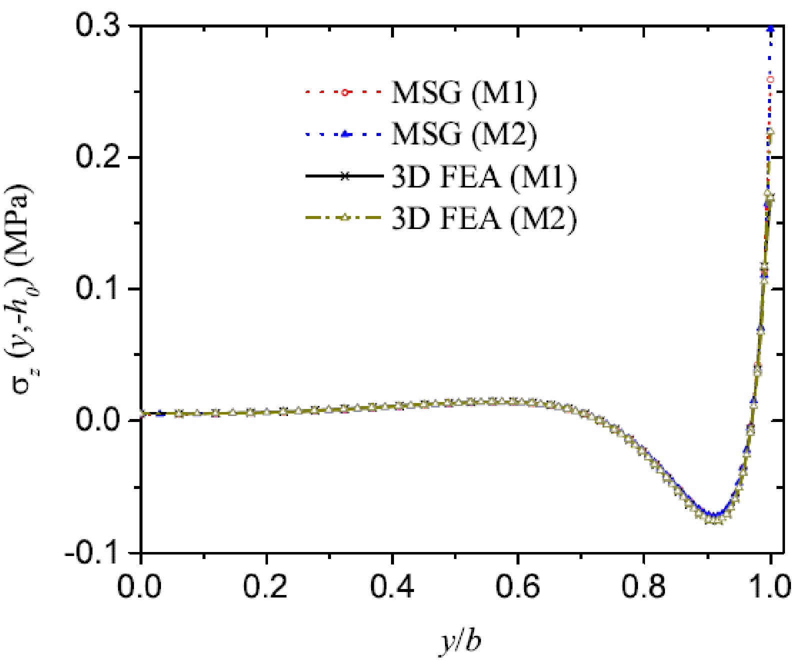


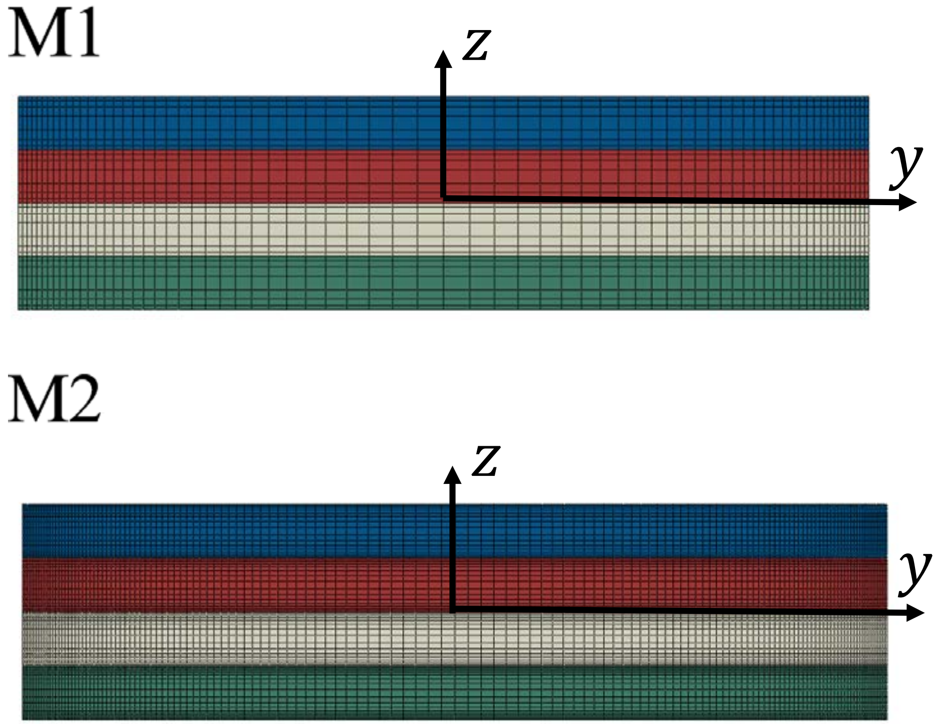




Combined Load on [60/30/-45/45]

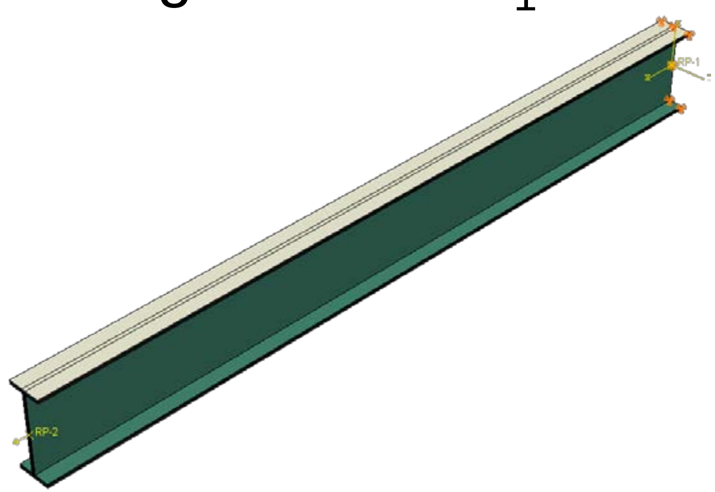
- Element: 4 nodes quadrilateral element

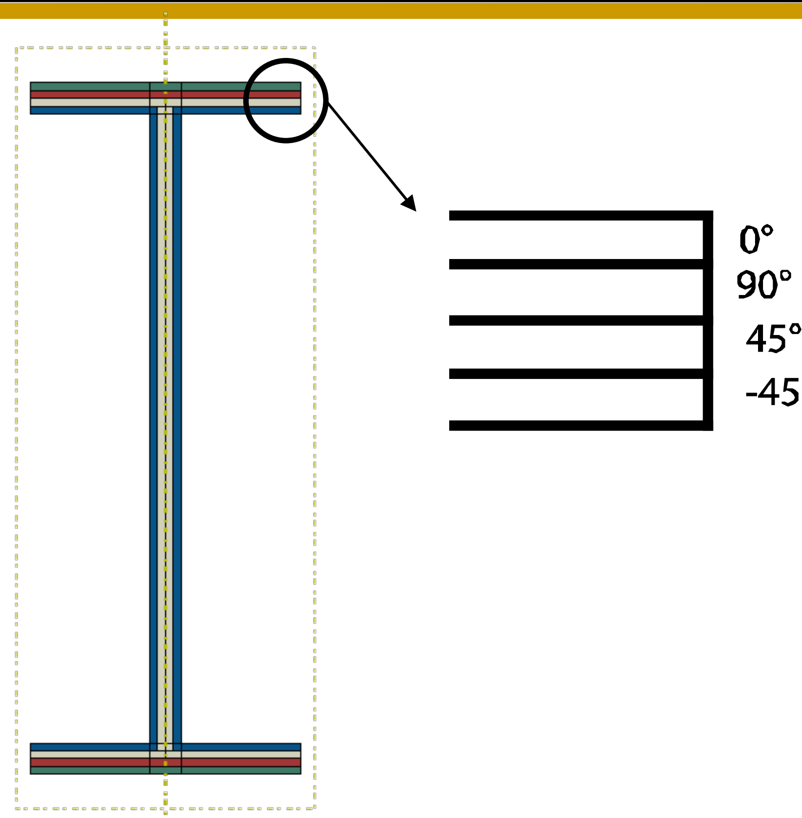




Various Loads on [0/90/45/-45] I-beam Laminate

- Boundary Conditions
  - Extension force F_1 = 100N
  - Bending moment M_{13} = 0.1N·m
  - Twisting moment M_1 = 0.1N·m

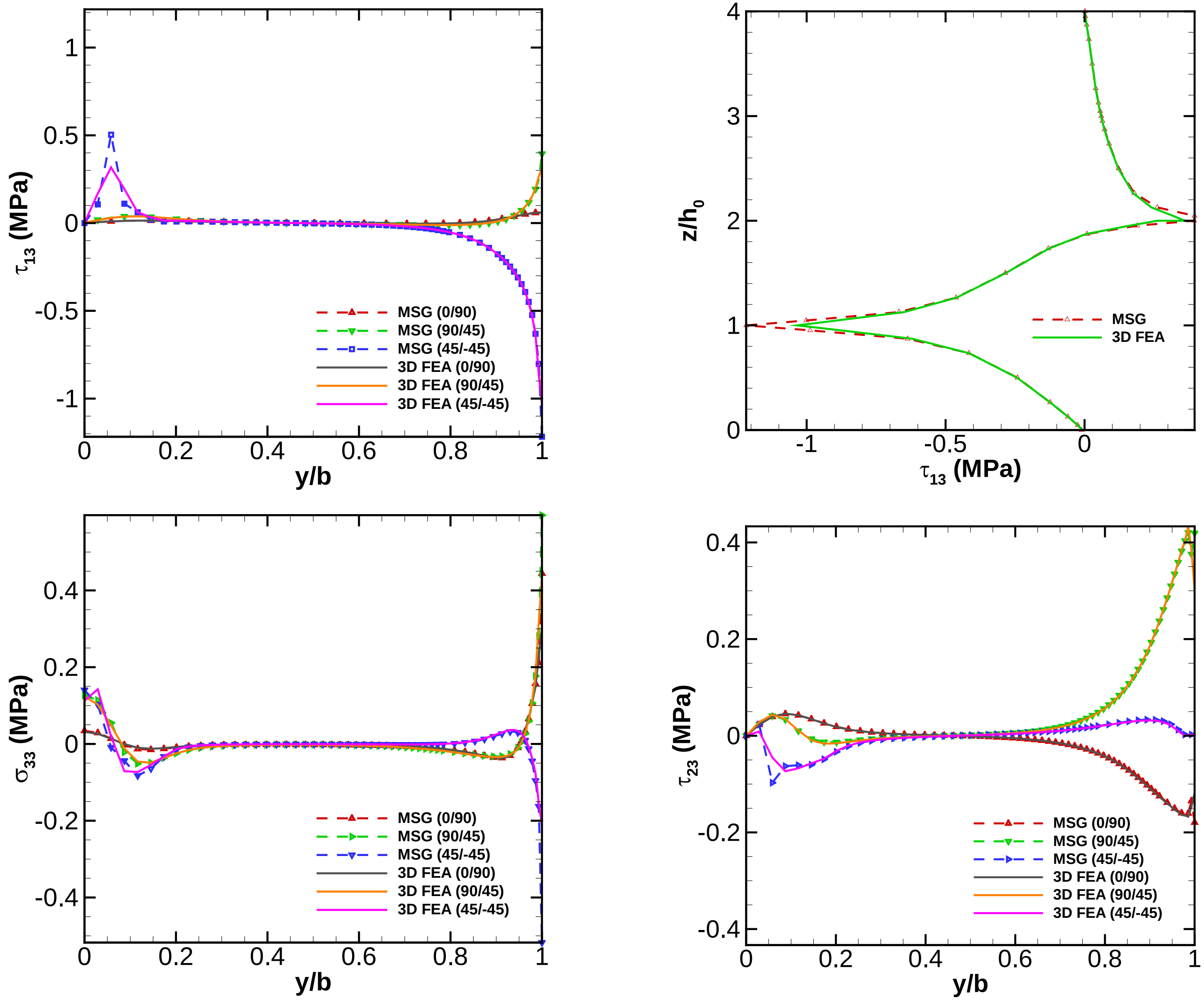




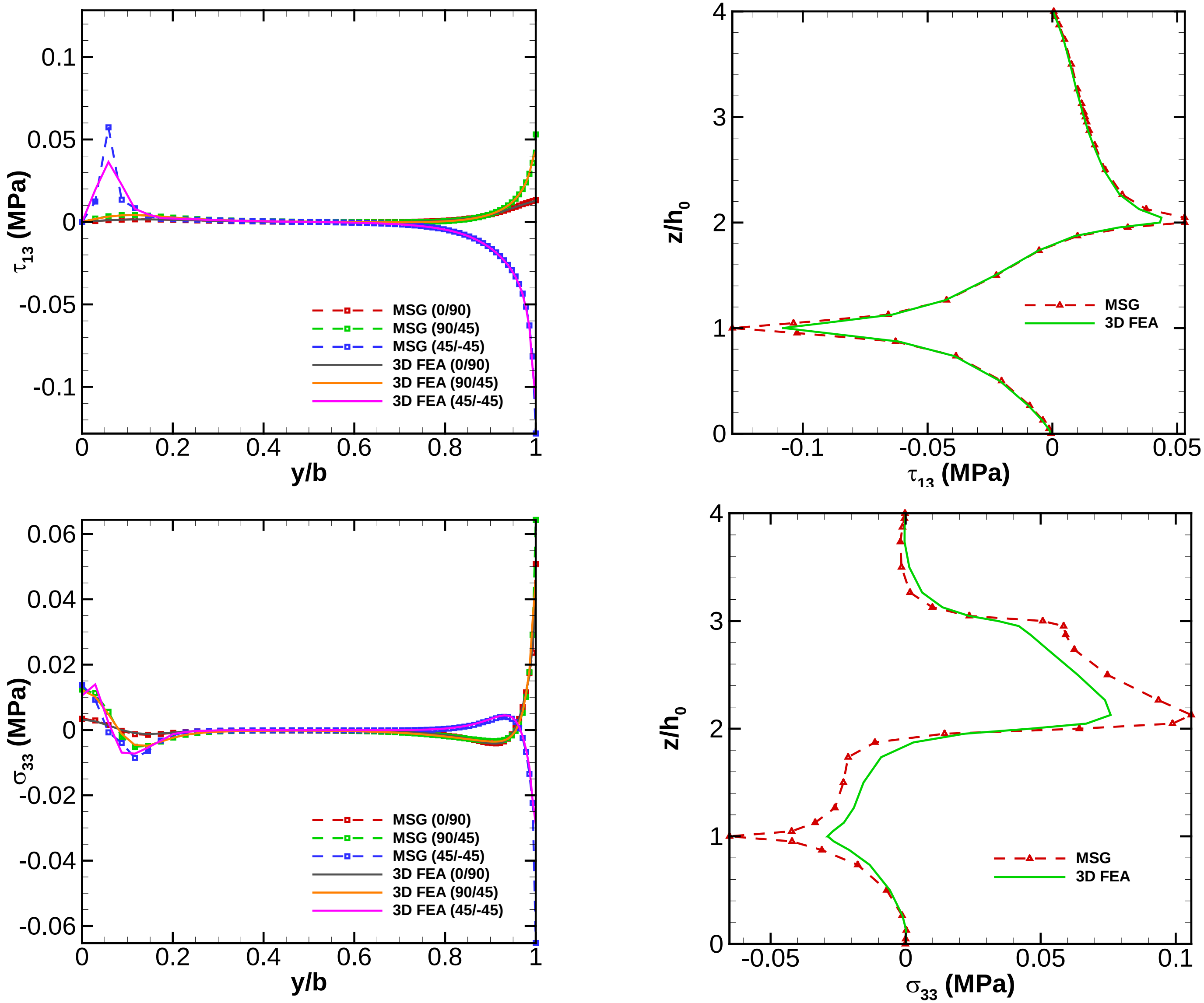
- Materials

E_1(MPa)	E_2(MPa)	E_3(MPa)	G_{12}(MPa)	G_{13}(MPa)	G_{23}(MPa)	v_{12}	v_{13}	v_{23}
132000	10800	10800	5650	5650	3380	0.24	0.24	0.59

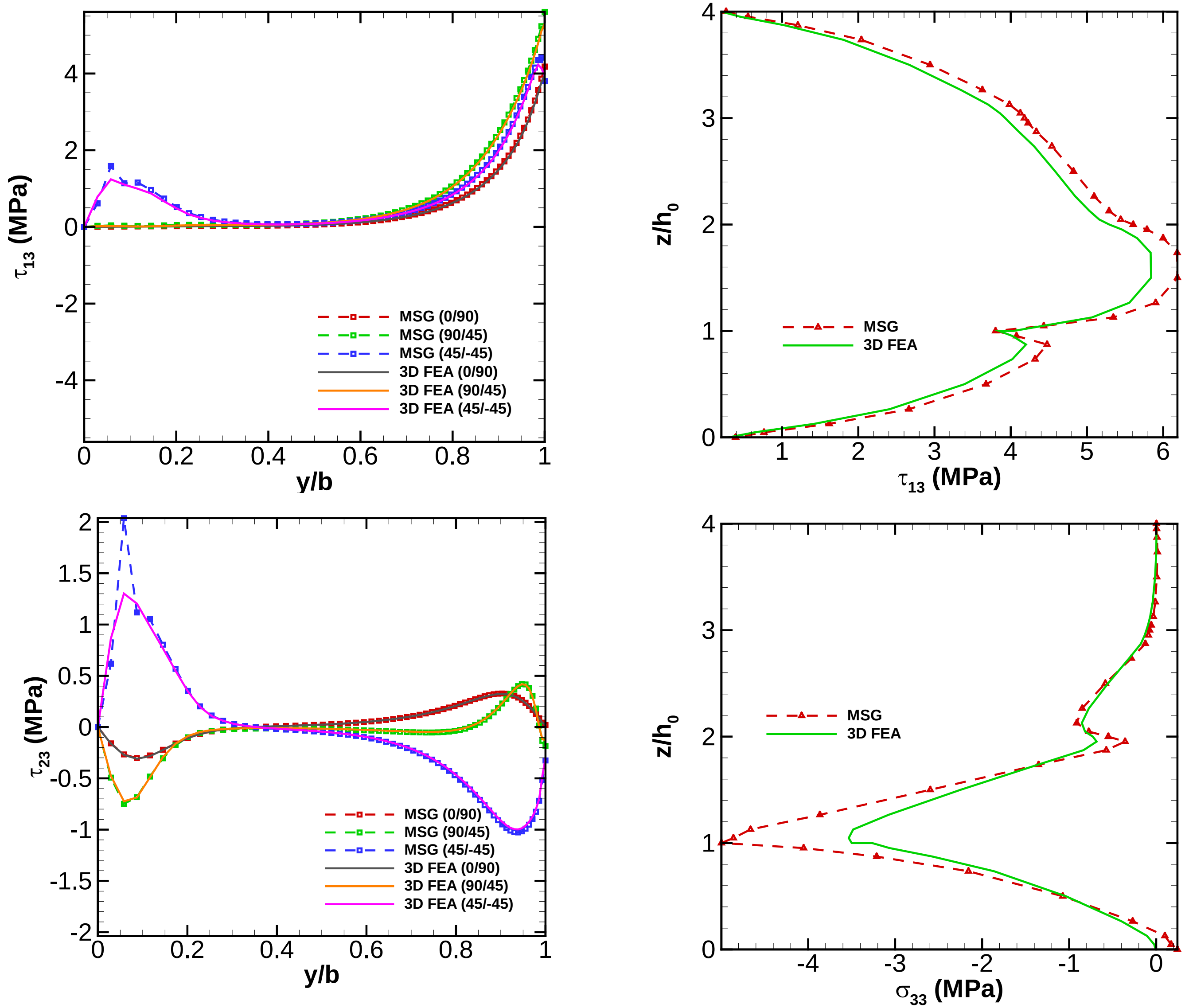
I-beam Laminate under Extension



I-beam Laminate under Bending

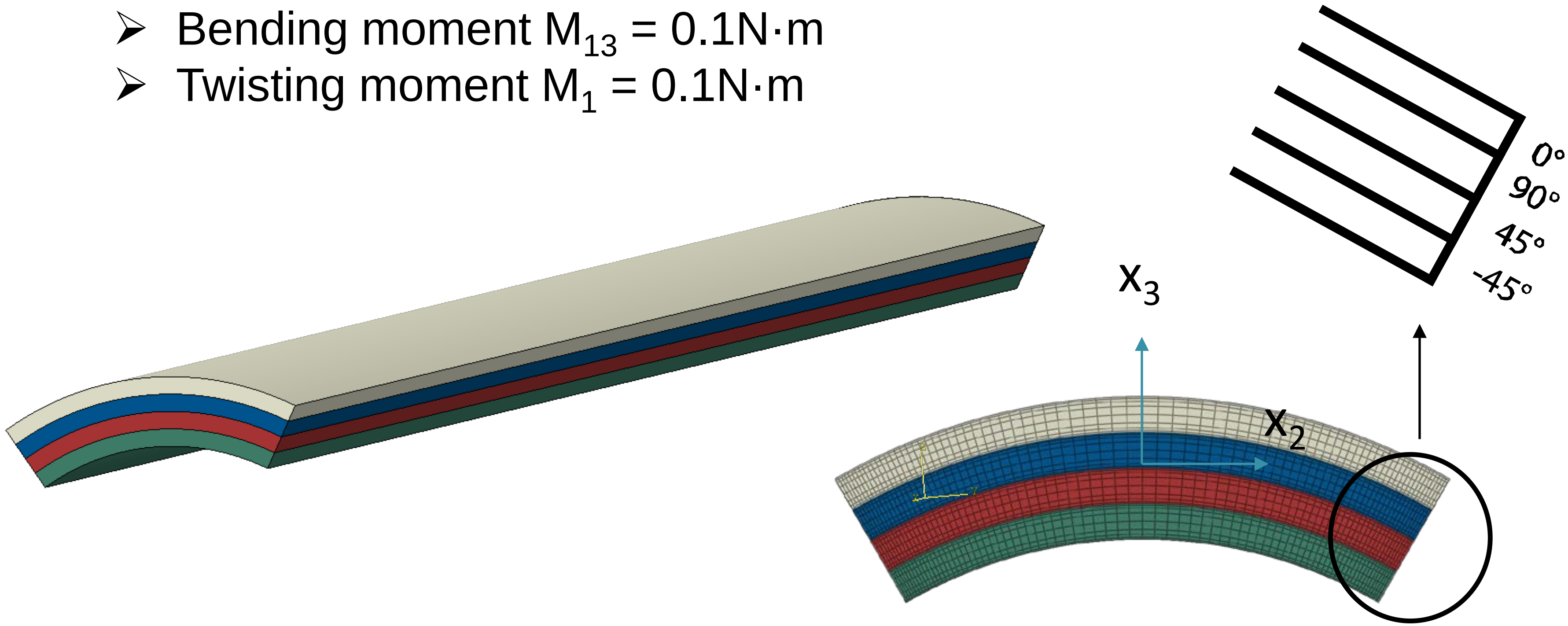


I-beam Laminate under Twisting



[0/90/45/-45] Laminate with Curved Cross Section

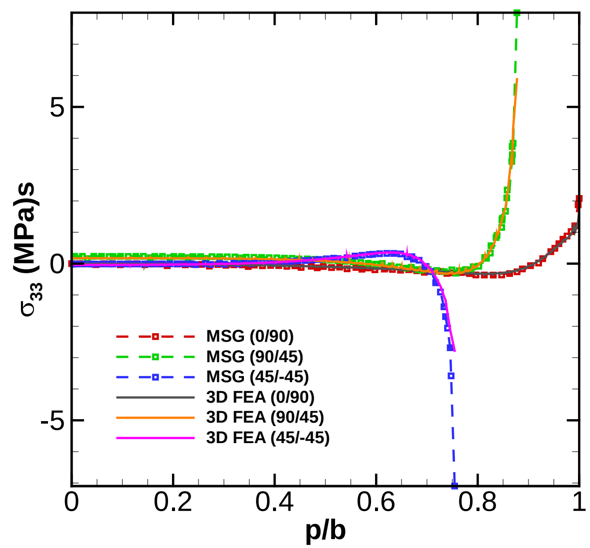
- Boundary Conditions
  - Extension force F_1 = 100N
  - Bending moment M_{13} = 0.1N·m
  - Twisting moment M_1 = 0.1N·m

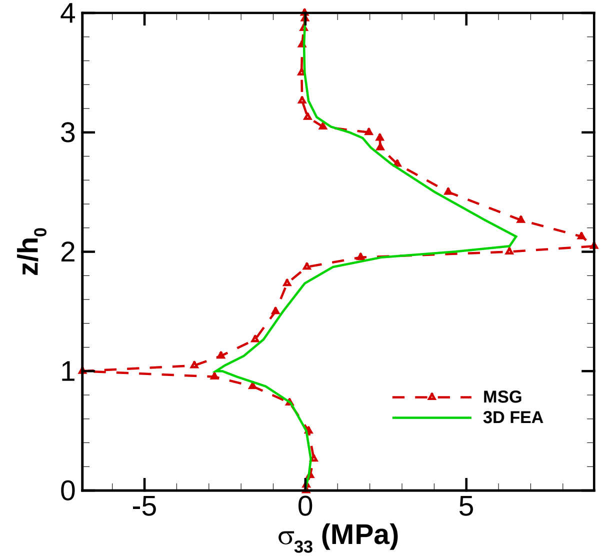


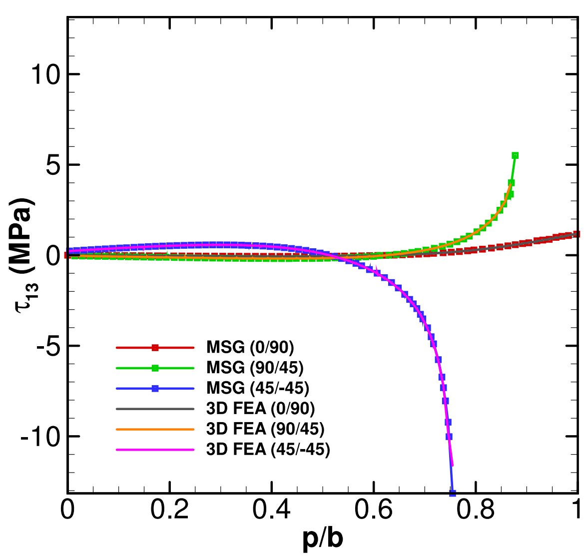
- Materials

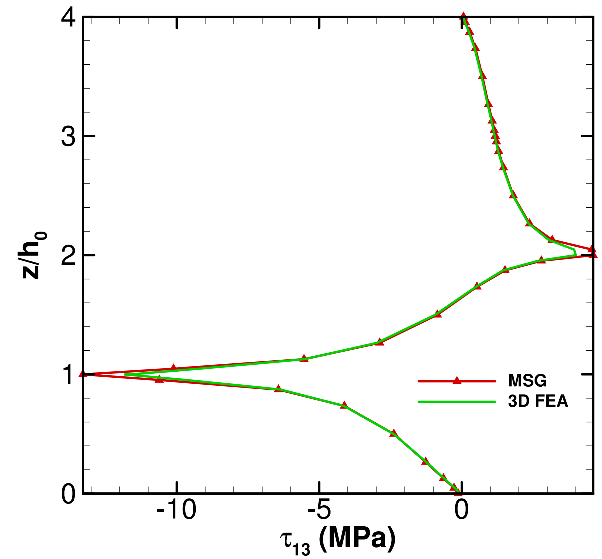
E_1(MPa)	E_2(MPa)	E_3(MPa)	G_{12}(MPa)	G_{13}(MPa)	G_{23}(MPa)	v_{12}	v_{13}	v_{23}
132000	10800	10800	5650	5650	3380	0.24	0.24	0.59

Laminate with Curved Cross Section under Extension

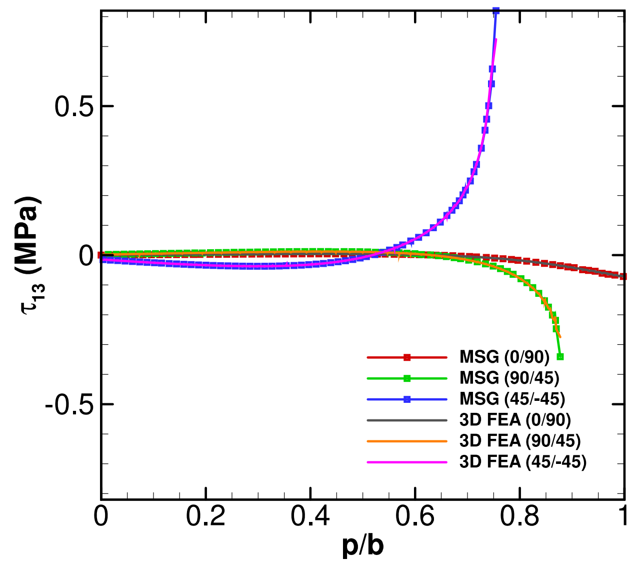


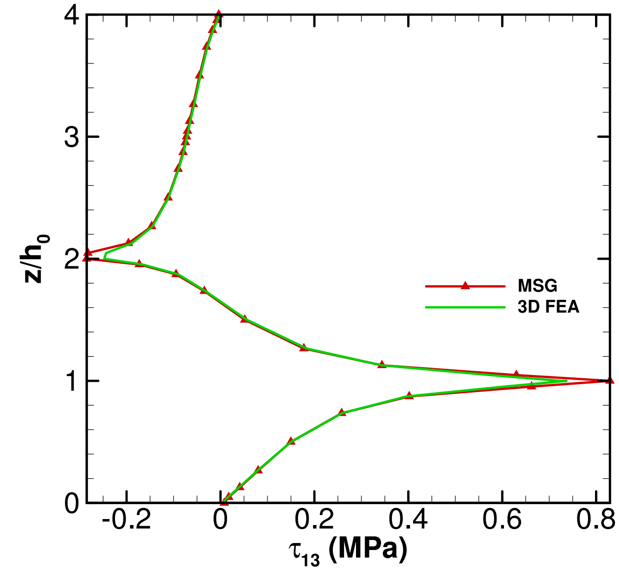


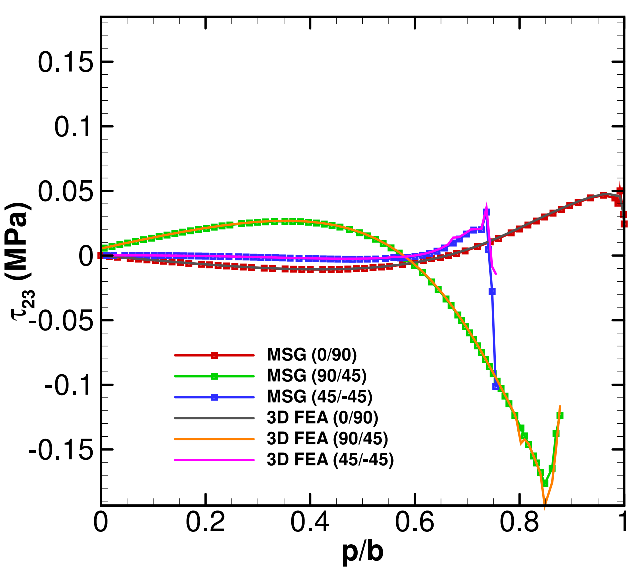


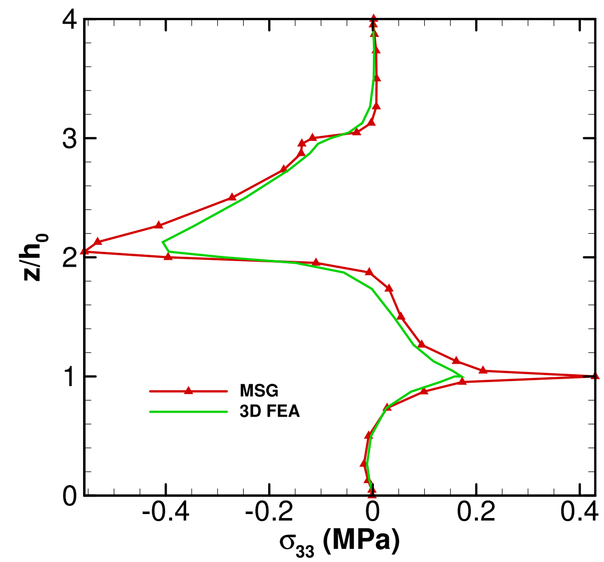


Laminate with Curved Cross Section under Bending

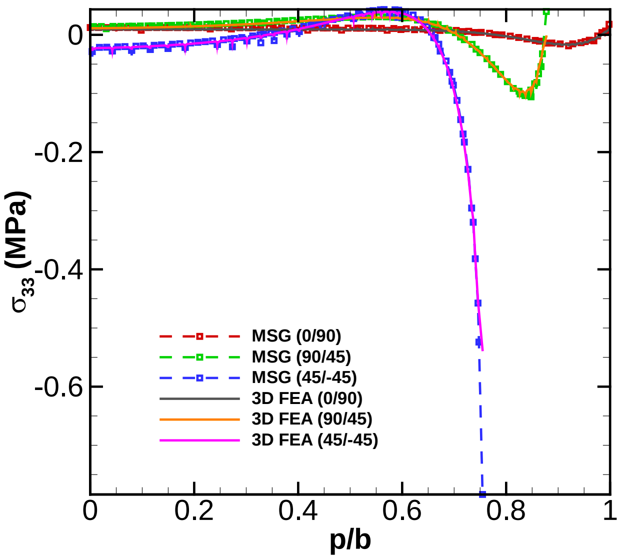


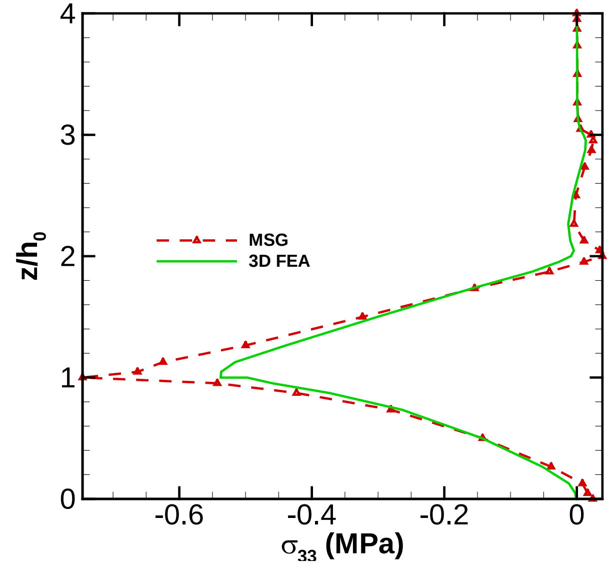


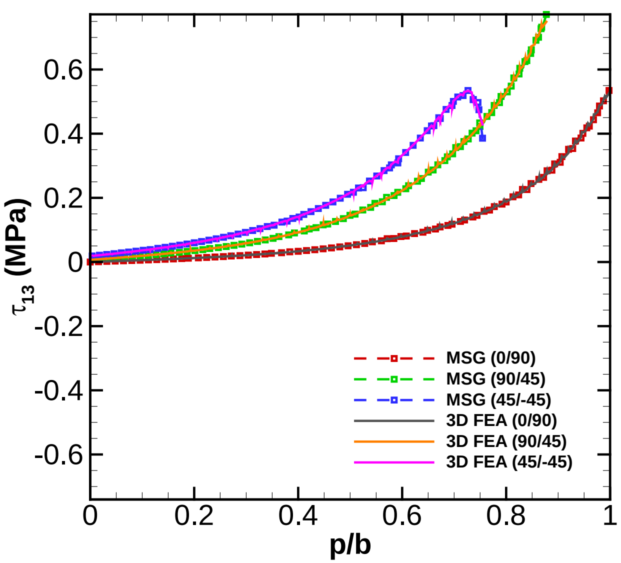


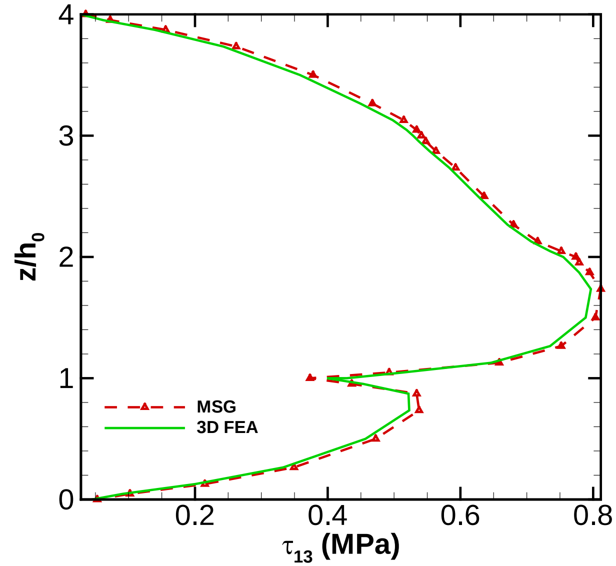


Laminate with Curved Cross Section under Twisting









Conclusions

- MSG cross-sectional analysis can be used to solve general free-edge stress problems of composite laminates.
- It does not require the laminate subjected to constant loads along the x direction.
- No ad hoc assumptions on displacement or stress.
- No restriction on the geometry of the cross section.
- Efficient but accurate compared with 3D FEA.
- Converge faster than 3D FEA due to it's semi-analytical nature.
- MSG is implemented in SwiftComp, which can be used as a general-purpose tool for free edge analysis.

Right Results
Right Away



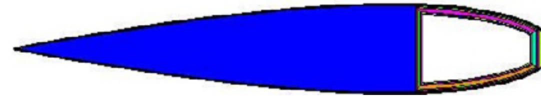
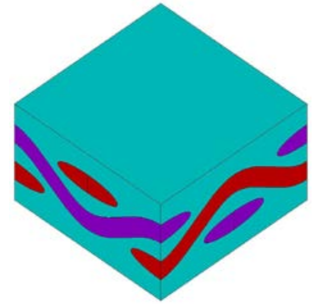
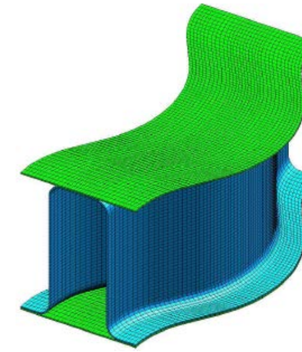
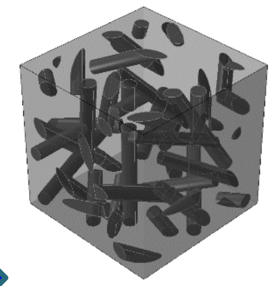
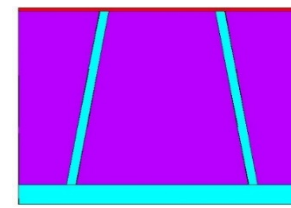
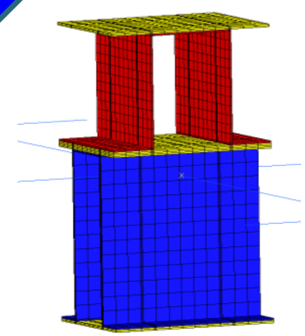
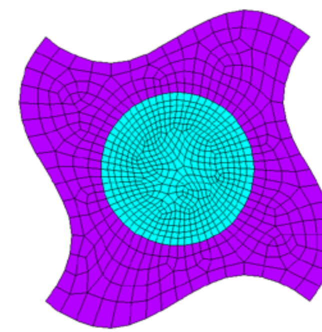
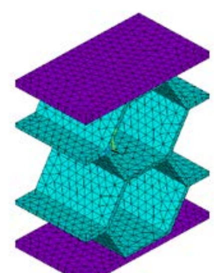
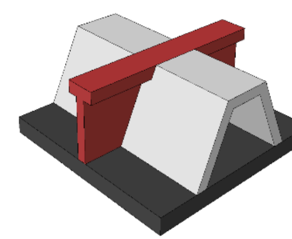

SwiftComp™

A Purdue Technology

Principle of Minimum Information Loss

- **Virtual testing of materials**
  - Mechanical properties
  - Multifunctional properties

- **Multiscale modeling of structures**
  - Composite laminates
  - Build-up structures: stiffened, sandwiched, corrugated

Acknowledgements

THE COMPOSITES iacmi INSTITUTE

Institute for ADVANCED Composites Manufacturing INNOVATION

Design, Modeling & Simulation Technology Area

Georgia Institute of Technology

U.S. ARMY

Army Vertical Lift Research Center of Excellence

AFRL THE AIR FORCE RESEARCH LABORATORY LEAD | DISCOVER | DEVELOP | DELIVER AFOSR

Multi-Scale Structural Mechanics and Prognosis