



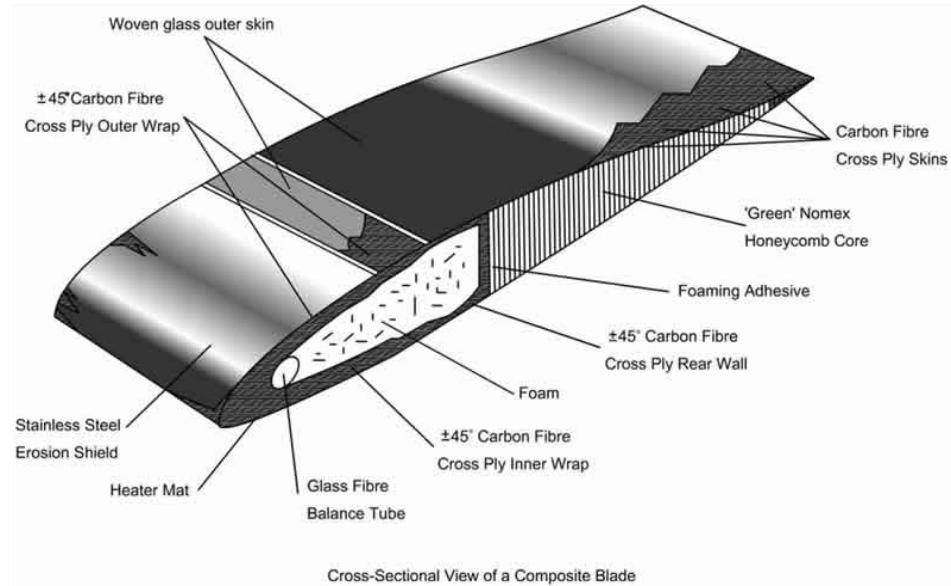
# POWERING SIMPLE BEAM ELEMENTS WITH DETAILED 3D FEA FIDELITY

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# The Problem



## 3D FEA

At least one solid elements/layer

$10^9$  DOFs/blade

Not suitable for design & optimization

**Smear property approach improves efficiency but loses significant accuracy**

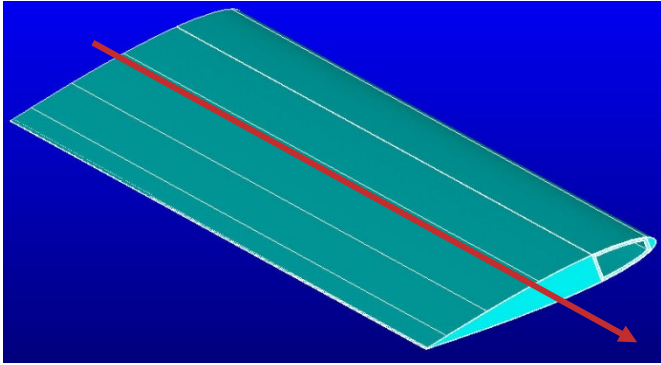
Length: 8.6 m

Chord: 0.72 m.

D-spar: 60 graphite/epoxy plies

Ply thickness: 125 microns

# VABS: Beam Constitutive Modeling



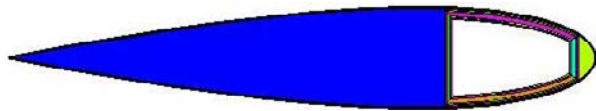
## 3D Elasticity

$$\sigma_{ij,j} + f_i = 0$$

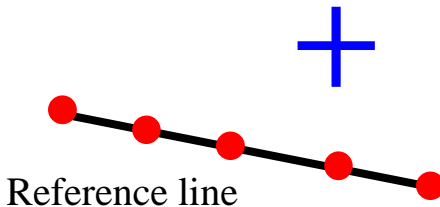
$$\varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i})$$

$$\begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{23} \\ \sigma_{13} \\ \sigma_{12} \end{Bmatrix} = \begin{bmatrix} c_{11} & c_{12} & c_{13} & c_{14} & c_{15} & c_{16} \\ c_{12} & c_{22} & c_{23} & c_{24} & c_{25} & c_{26} \\ c_{13} & c_{23} & c_{33} & c_{34} & c_{35} & c_{36} \\ c_{14} & c_{24} & c_{34} & c_{44} & c_{45} & c_{46} \\ c_{15} & c_{25} & c_{35} & c_{45} & c_{55} & c_{56} \\ c_{16} & c_{26} & c_{36} & c_{46} & c_{56} & c_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ 2\varepsilon_{23} \\ 2\varepsilon_{13} \\ 2\varepsilon_{12} \end{Bmatrix}$$

Reference line



$$\begin{Bmatrix} F_1 \\ M_1 \\ M_2 \\ M_3 \end{Bmatrix} = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{12} & S_{22} & S_{23} & S_{24} \\ S_{13} & S_{23} & S_{33} & S_{34} \\ S_{14} & S_{24} & S_{34} & S_{44} \end{bmatrix} \begin{Bmatrix} \gamma_{11} \\ \kappa_1 \\ \kappa_2 \\ \kappa_3 \end{Bmatrix}$$



1D beam analysis

$$\gamma_{11} = \bar{u}'_1$$

$$\kappa_1 = \Phi'_1$$

$$\kappa_2 = -\bar{u}''_3$$

$$\kappa_3 = \bar{u}''_2$$

$$\frac{dF_1}{dx_1} + p_1 = 0$$

$$\frac{dM_1}{dx_1} + q_1 = 0$$

$$\frac{d^2 M_2}{dx_1^2} + p_3 + \frac{dq_2}{dx_1} = 0$$

$$\frac{d^2 M_3}{dx_1^2} - p_2 + \frac{dq_3}{dx_1} = 0$$

# Introduction to VABS Theory

- Minimize kinetic energy loss.
- A diagonal mass matrix is possible if blade axis is at the mass center and sectional coordinates are the principal inertia axes.

$$\mathcal{K} = \frac{1}{2} \begin{Bmatrix} V_1 \\ V_2 \\ V_3 \\ \Omega_1 \\ \Omega_2 \\ \Omega_3 \end{Bmatrix}^T \begin{bmatrix} \mu & 0 & 0 & 0 & \mu x_{m3} & -\mu x_{m2} \\ & \mu & 0 & -\mu x_{m3} & 0 & 0 \\ & & \mu & \mu x_{m2} & 0 & 0 \\ & & & i_{22} + i_{33} & 0 & 0 \\ & \text{symmetric} & & & i_{22} & -i_{23} \\ & & & & & i_{33} \end{bmatrix} \begin{Bmatrix} V_1 \\ V_2 \\ V_3 \\ \Omega_1 \\ \Omega_2 \\ \Omega_3 \end{Bmatrix}$$

# Introduction to VABS Theory

➤ Euler-Bernoulli model

$$\begin{Bmatrix} F_1 \\ M_1 \\ M_2 \\ M_3 \end{Bmatrix} = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{12} & S_{22} & S_{23} & S_{24} \\ S_{13} & S_{23} & S_{33} & S_{34} \\ S_{14} & S_{24} & S_{34} & S_{44} \end{bmatrix} \begin{Bmatrix} \gamma_{11} \\ \kappa_1 \\ \kappa_2 \\ \kappa_3 \end{Bmatrix}$$

➤ Timoshenko model

$$\begin{Bmatrix} F_1 \\ F_2 \\ F_3 \\ M_1 \\ M_2 \\ M_3 \end{Bmatrix} = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} & S_{15} & S_{16} \\ S_{12} & S_{22} & S_{23} & S_{24} & S_{25} & S_{26} \\ S_{13} & S_{23} & S_{33} & S_{34} & S_{35} & S_{36} \\ S_{14} & S_{24} & S_{34} & S_{44} & S_{45} & S_{46} \\ S_{15} & S_{25} & S_{35} & S_{45} & S_{55} & S_{56} \\ S_{16} & S_{26} & S_{36} & S_{46} & S_{56} & S_{66} \end{bmatrix} \begin{Bmatrix} \gamma_{11} \\ 2\gamma_{12} \\ 2\gamma_{13} \\ \kappa_1 \\ \kappa_2 \\ \kappa_3 \end{Bmatrix}$$

➤ 1D beam analysis should be changed to allow fully populated stiffness matrices.

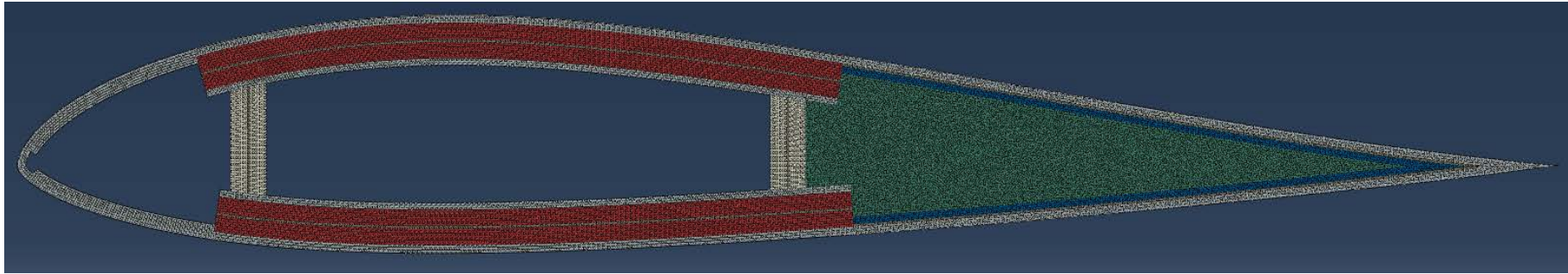
# Introduction to VABS Theory

- Vlasov model: important for thin-walled beams with open sections

$$\begin{Bmatrix} F_1 \\ M_1 \\ M_2 \\ M_3 \\ M_\omega \end{Bmatrix} = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} & S_{15} \\ S_{12} & S_{22} & S_{23} & S_{24} & S_{25} \\ S_{13} & S_{23} & S_{33} & S_{34} & S_{35} \\ S_{14} & S_{24} & S_{34} & S_{44} & S_{45} \\ S_{15} & S_{25} & S_{35} & S_{45} & S_{55} \end{bmatrix} \begin{Bmatrix} \gamma_{11} \\ \kappa_1 \\ \kappa_2 \\ \kappa_3 \\ \kappa'_1 \end{Bmatrix}$$

- VABS can also
  - Deal with trapeze effect, oblique sections
  - Locate neutral axis, principal bending/inertia axes, shear center
  - Recover 3D displacement/strain/stress
  - Model beams made of smart materials (coupled thermo-elasto-electro-magnetic behavior)

# What Can VABS Do for You?



- **VABS** takes a finite element discretization of sectional geometry and material as input to calculate sectional properties, which are needed for **any** beam analysis code to predict global behavior. VABS also recovers 3D displacements/strains/stresses over the section.
- **VABS** can be used independently for **structural design of beam sections** (topology and material): e.g., maximize torsional stiffness while maintain desired center of gravity.
- **VABS** powers conventional beam elements with the fidelity of 3D detailed FEA for geometry representation and prediction with negligible additional computing time.

# VABS: Efficient High-Fidelity Modeling of Composite Slender Structures

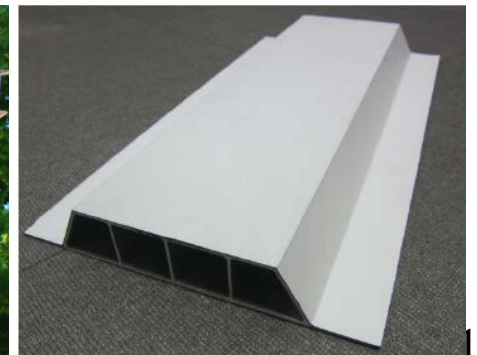
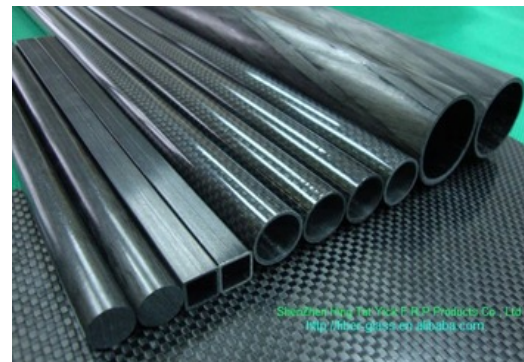
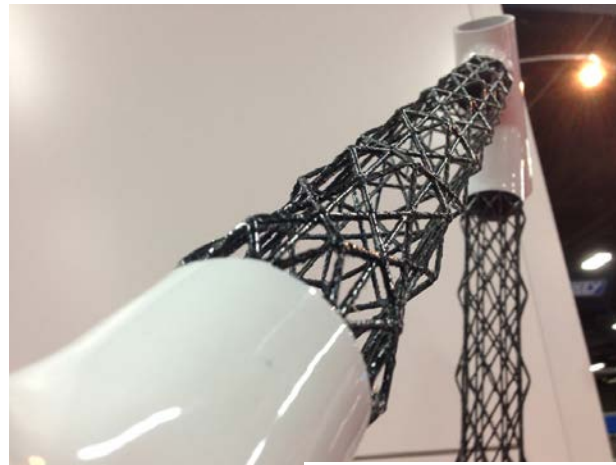
- A unique technology continuously funded by US Army since 1988 (28 yrs, 3 more to come).
- *VABS as a Tool for Real Blade Analysis, Design, & Certification*, Task 3 of Georgia Tech Vertical Lift Center of Excellence (2017-2019).
- Tool of choice for helicopter industry and wind turbine industry.
- Efficient high-fidelity solutions for slender parts: one dimension  $\gg$  two other dimensions



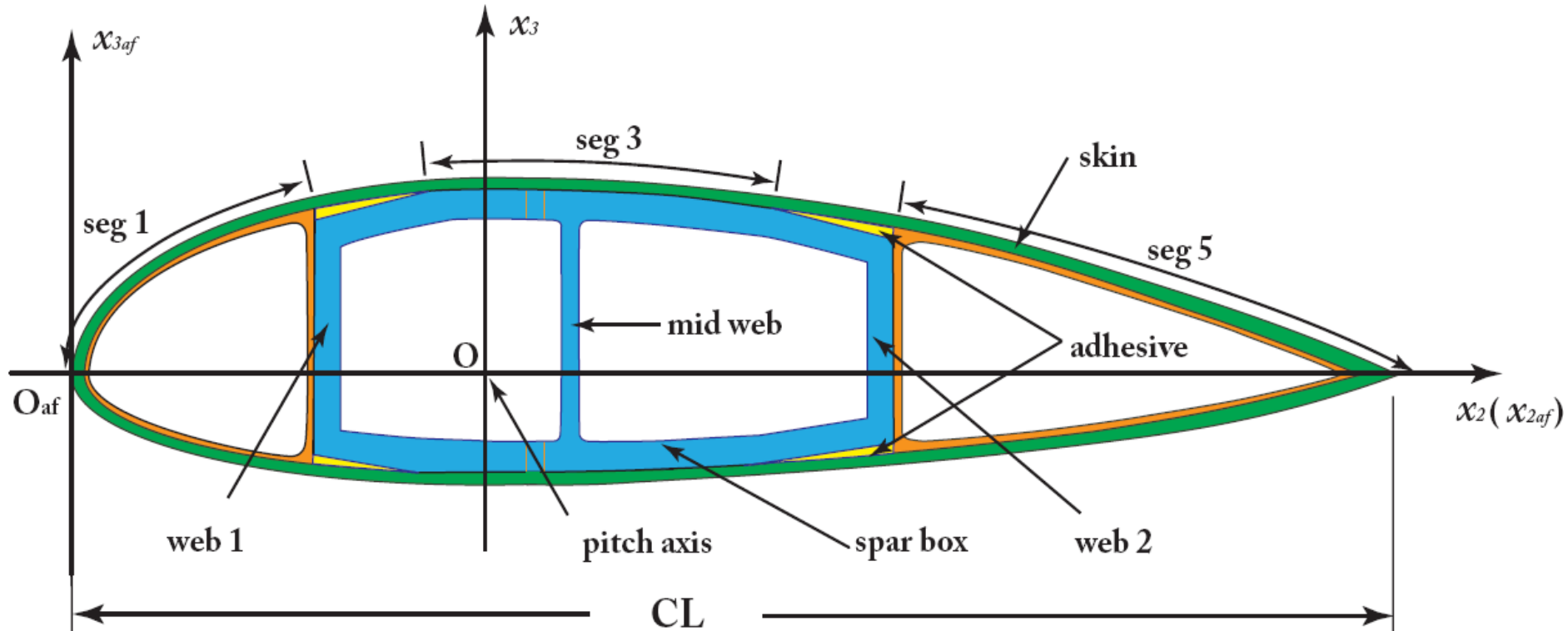
# Related Problems



# Related Problems



# Properties of a Wind Turbine Blade

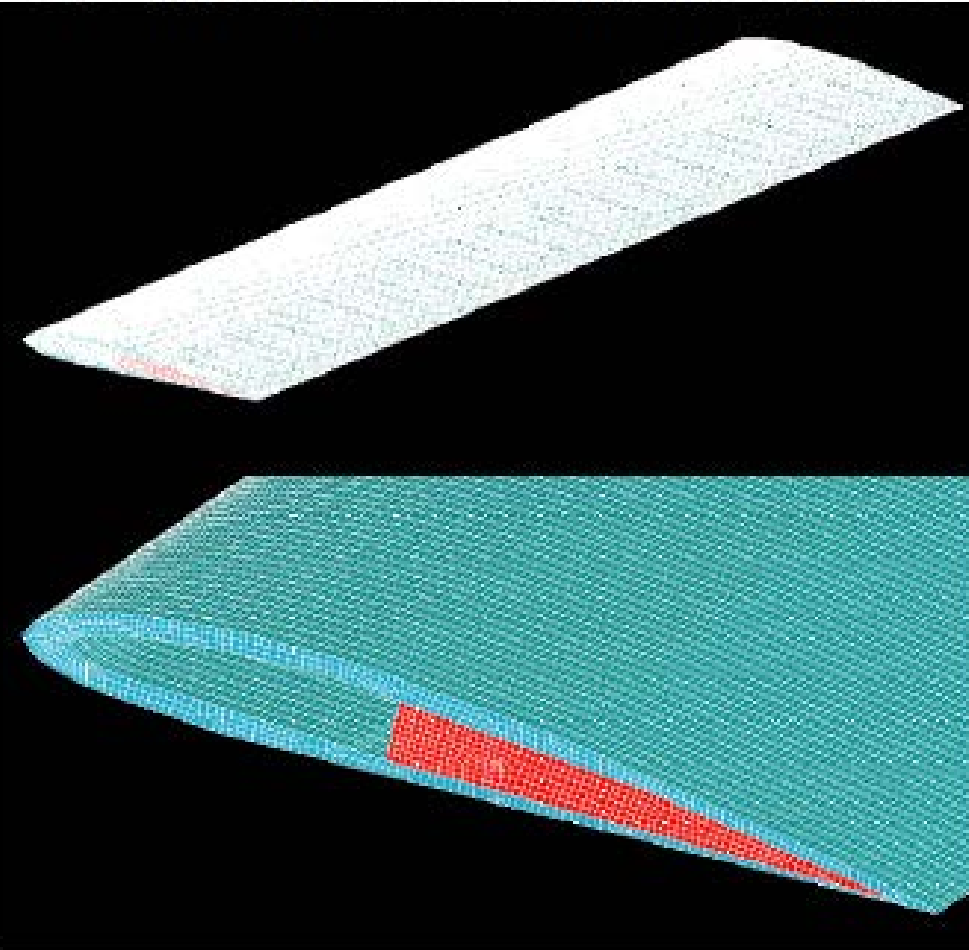


Chen, H. and Yu, W. and Capellaro, M.: “A Critical Assessment of Computer Tools for Calculating Composite Wind Turbine Blade Properties,” *Wind Energy*, vol. 13, no. 6, 2010, pp. 497-516.

# Properties of a Wind Turbine Blade

	PreComp	CROSTAB	VABS	% Diff. (PreComp)	% Diff. (CROSTAB)
$EI_{22}$	2.103E+07	1.459E+08	1.916E+07	9.778	661.734
$EI_{33}$	6.309E+08	4.878E+08	4.398E+08	43.448	10.907
$GJ$	1.008E+07	2.469E+07	2.167E+07	53.479	13.950
$EA$	3.000E+09	2.789E+09	2.387E+09	25.664	16.826
$S_{34}$	-8.132E+06	6.010E+07	1.210E+07	167.204	396.632
$S_{13}$	-1.037E+06	5.216E+08	-2.635E+07	96.065	2.079E+03
$S_{14}$	-1.301E+08	1.685E+08	-4.724E+08	72.459	135.671
$S_{23}$	-3.776E+05	9.002E+09	-5.222E+04	623.105	1.724E+07
$S_{24}$	8.746E+06	-1.208E+09	1.422E+06	514.904	8.504E+04
$S_{12}$	7.522E+05	-1.723E+09	-3.381E+07	102.225	4.996E+03
$\mu$	285.9	289.132	258.053	10.791	12.044
$i_{22}$	2.211	5.144	2.172	1.797	136.837
$i_{33}$	62.72	61.340	46.418	35.121	32.148
$x_{m2}$	0.332	0.284	0.27780	19.444	2.064
$x_{m3}$	0.027	-0.028	0.02743	1.572	201.272
$x_{t2}$	0.331	-0.0290	0.233	42.173	112.466
$x_{t3}$	0.028	0.2273	0.029	3.287	685.174
$x_{s2}$	0.287	/	0.031	813.479	/
$x_{s3}$	0.028	/	0.040	30.478	/
$\theta$	-0.990	3.7919	-1.244	20.419	404.813

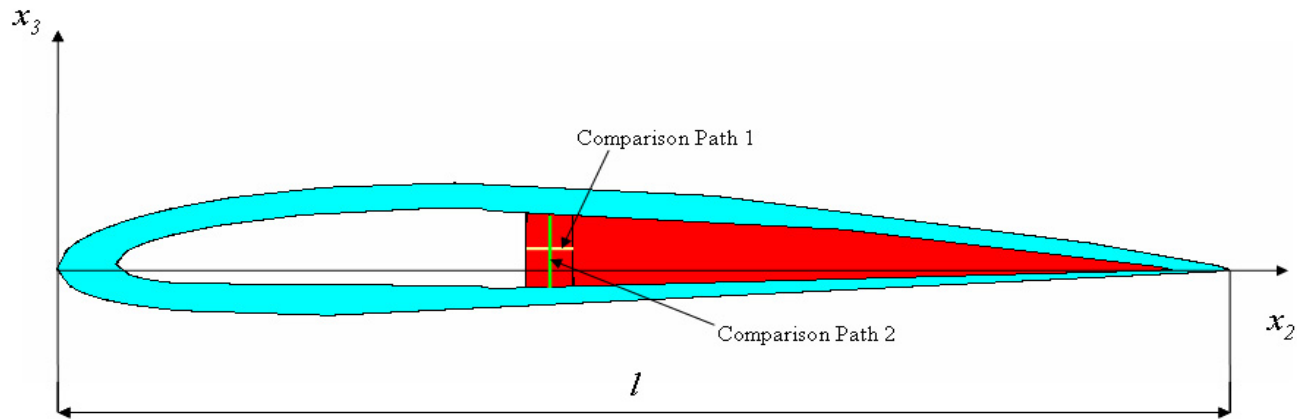
# Realistic Rotor Blade



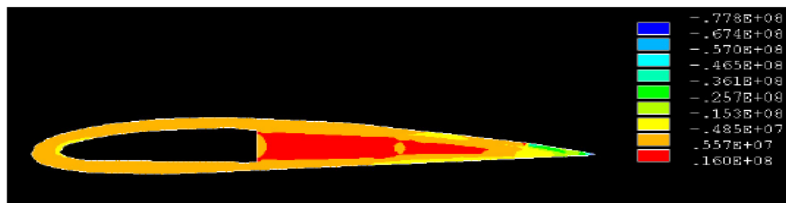
- Realistic rotor blade
- 100°C temperature increase
- Find thermal stress
- ANSYS model using brick elements (4.8M DOFs)

Wang, Q. and Yu, W.: “A Variational Asymptotic Approach for Thermoelastic Analysis of Composite Beams,” *Advances in Aircraft and Spacecraft Science*, vol. 1, 2014, pp. 93-123.

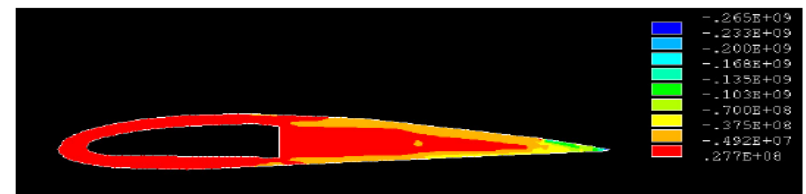
# Realistic Rotor Blade



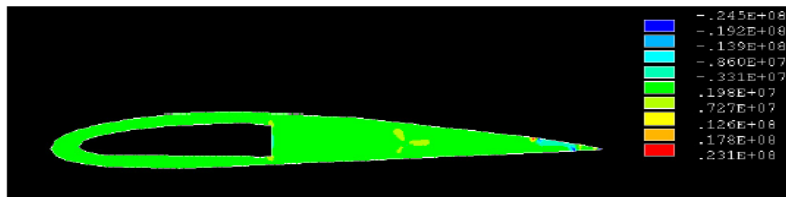
•  $\sigma_{11}$



•  $\sigma_{22}$



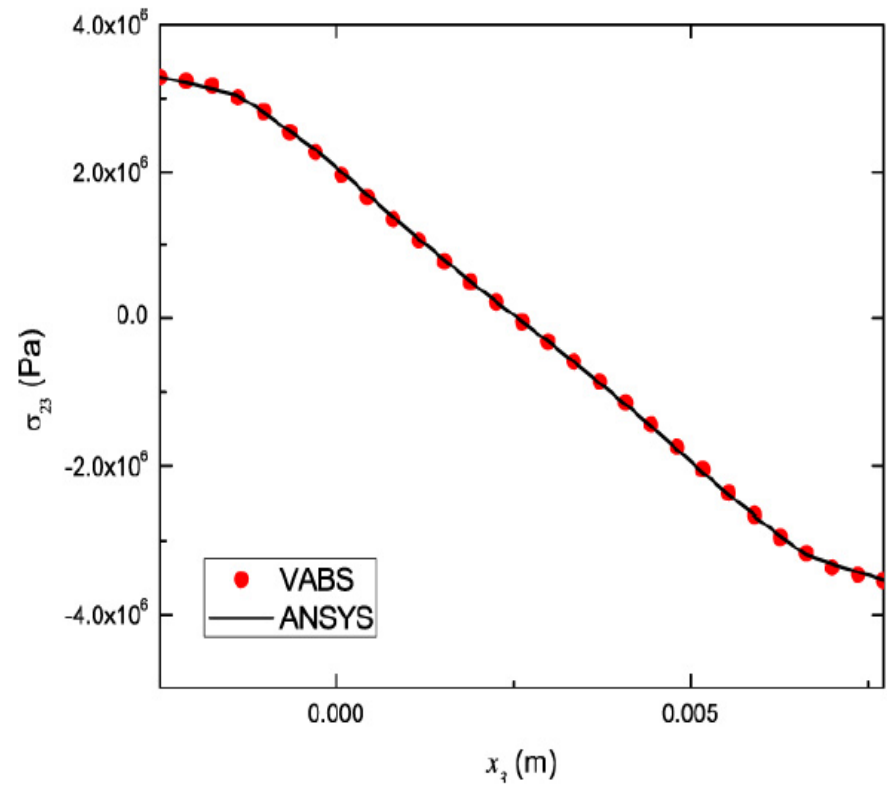
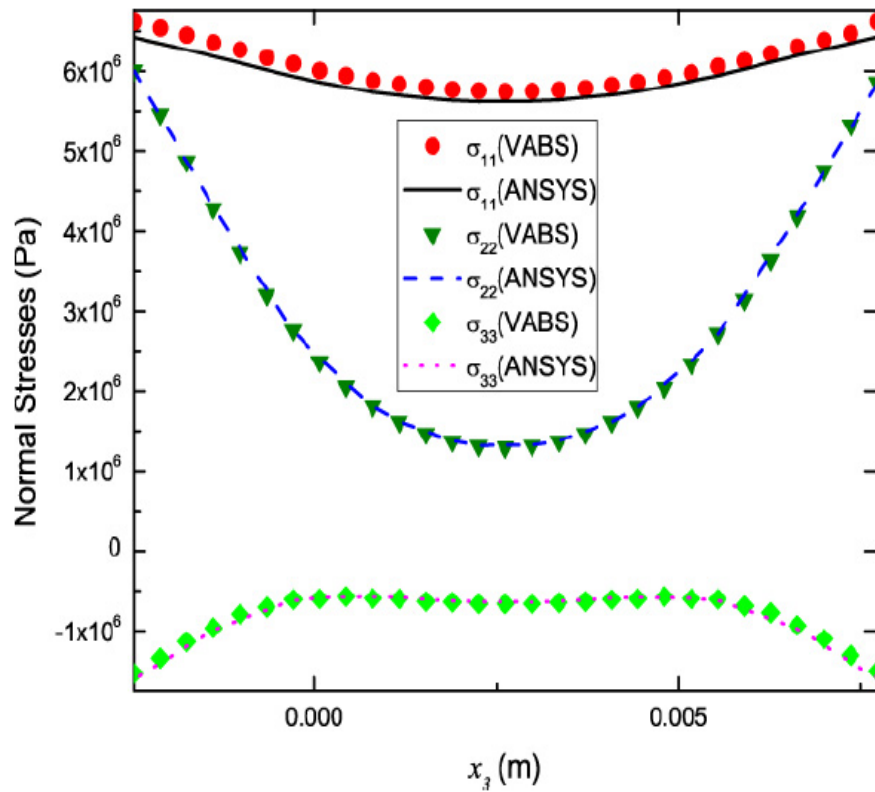
•  $\sigma_{33}$



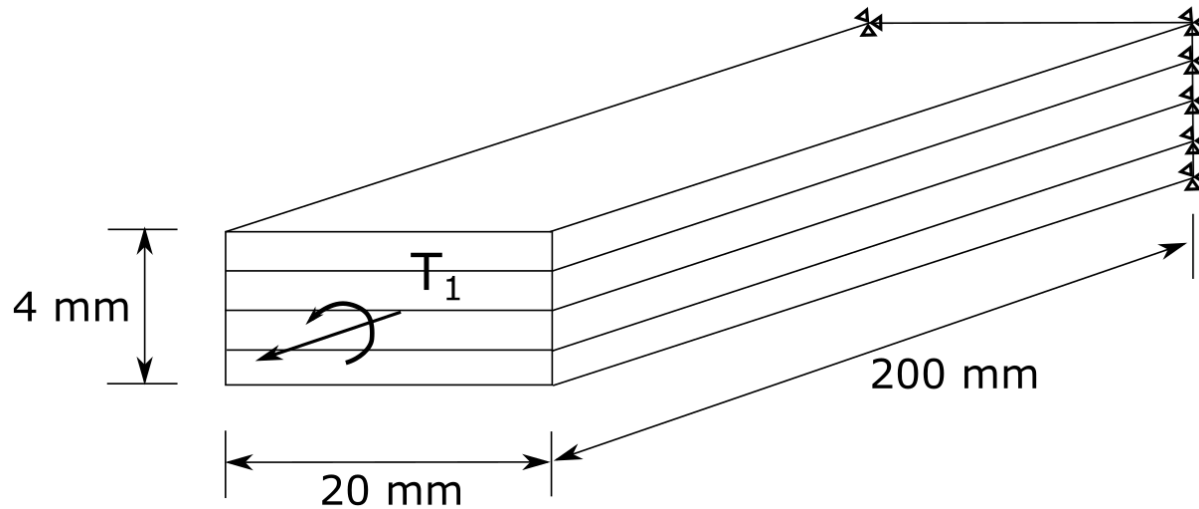
•  $\sigma_{23}$



# Realistic Rotor Blade



# 4-Layer Laminate Under Torque



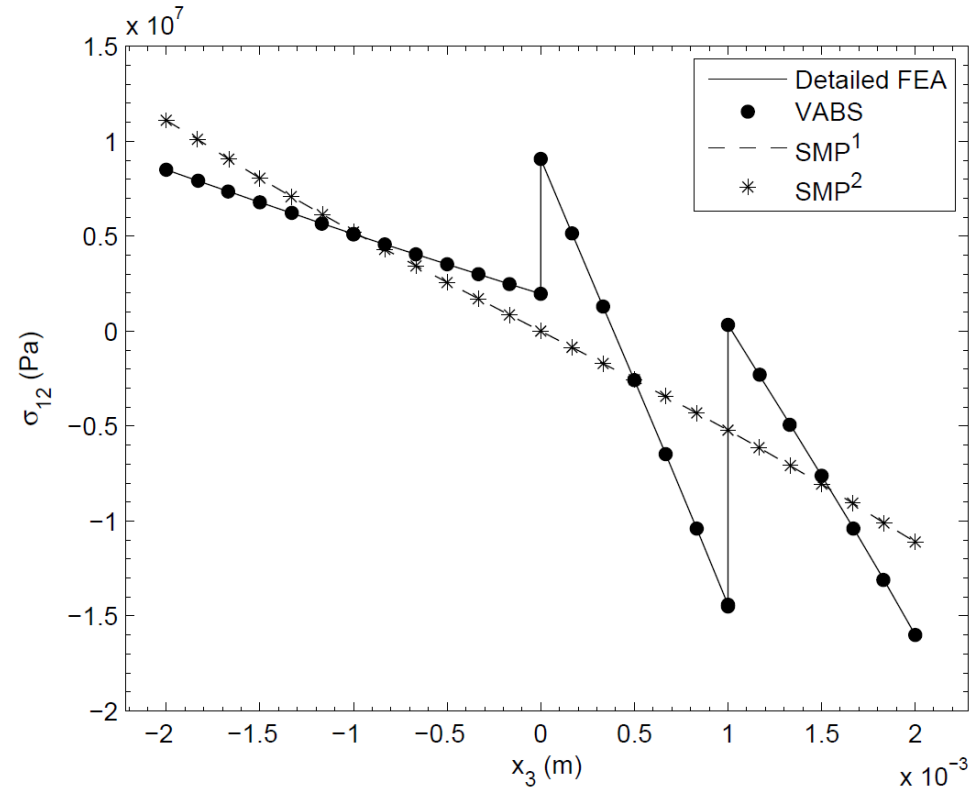
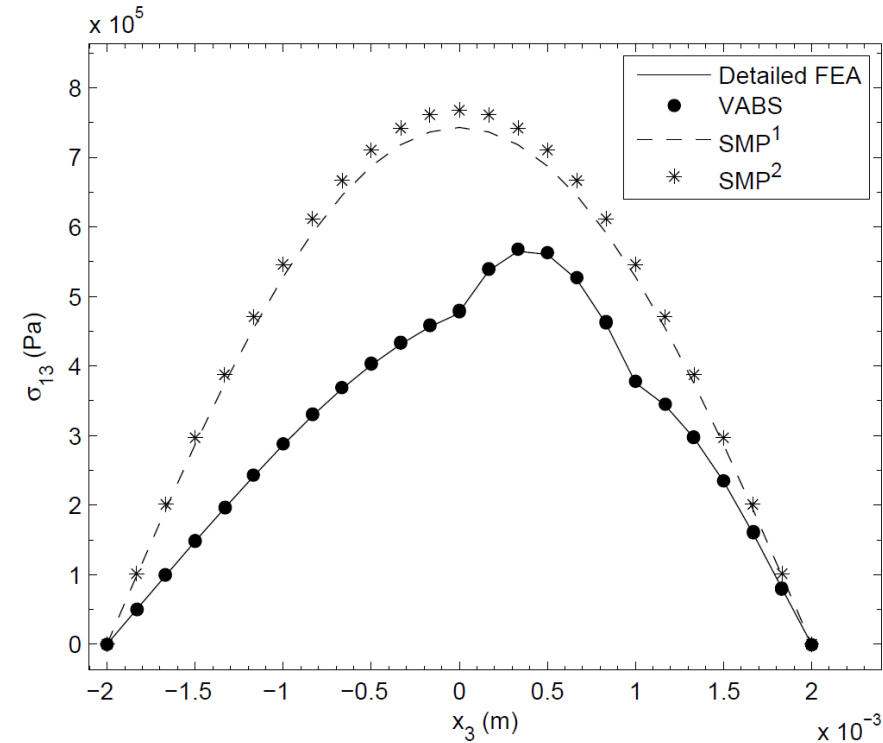
$$T_1 = 1N \cdot m$$

Layup Sequence	$E_{11}$ (GPa)	$E_{22}$ (GPa)	$E_{33}$ (GPa)	$G_{12}$ (GPa)	$G_{13}$ (GPa)	$G_{23}$ (GPa)	$\nu_{12}$	$\nu_{13}$	$\nu_{23}$
[0/90/±45]	132	10.8	10.8	5.65	5.65	3.38	0.24	0.24	0.59

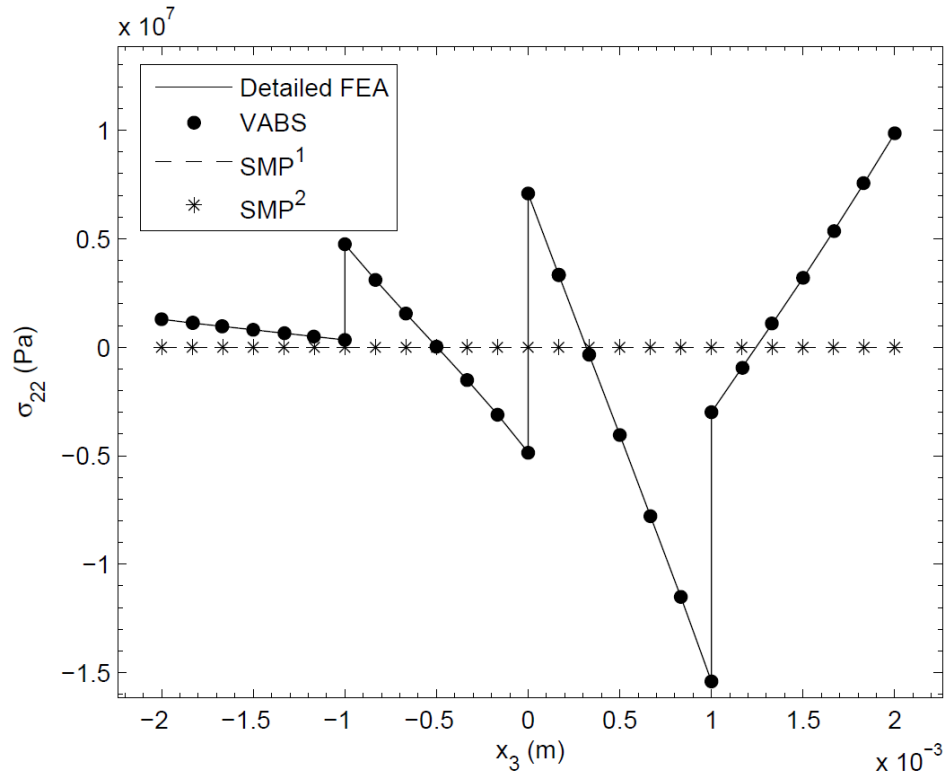
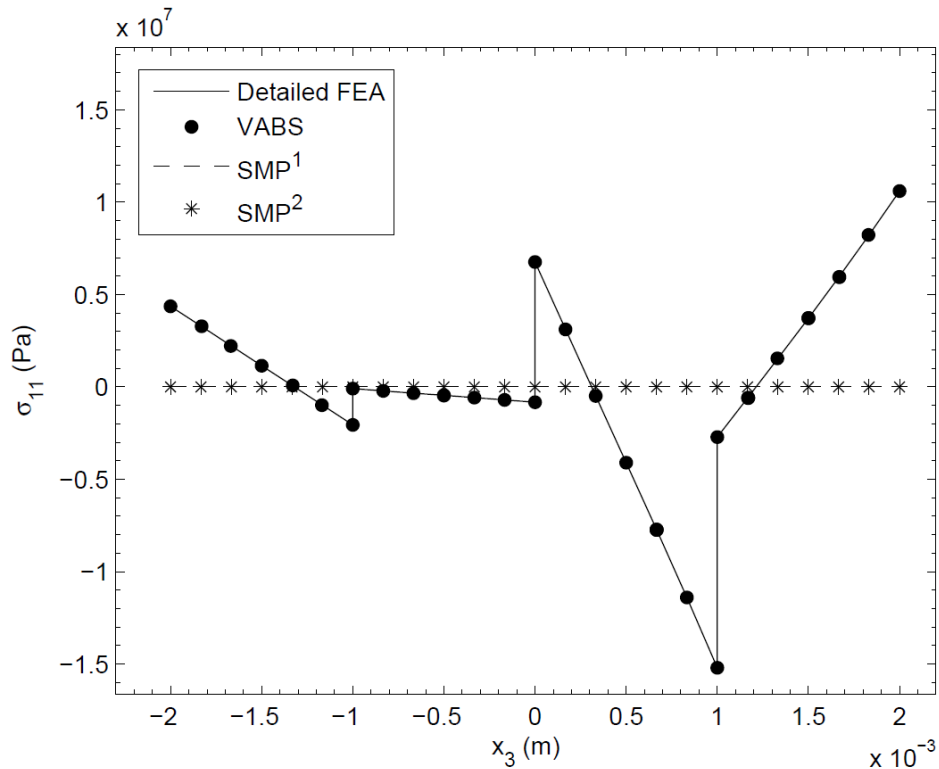
Liu, N. and Yu, W.: “Loss of Accuracy Using Smearred Properties in Composite Beam Modeling,” *Proceedings of the American Society for Composites 30<sup>th</sup> Technical Conference*, East Lansing, Michigan, Sept. 28-30, 2015.



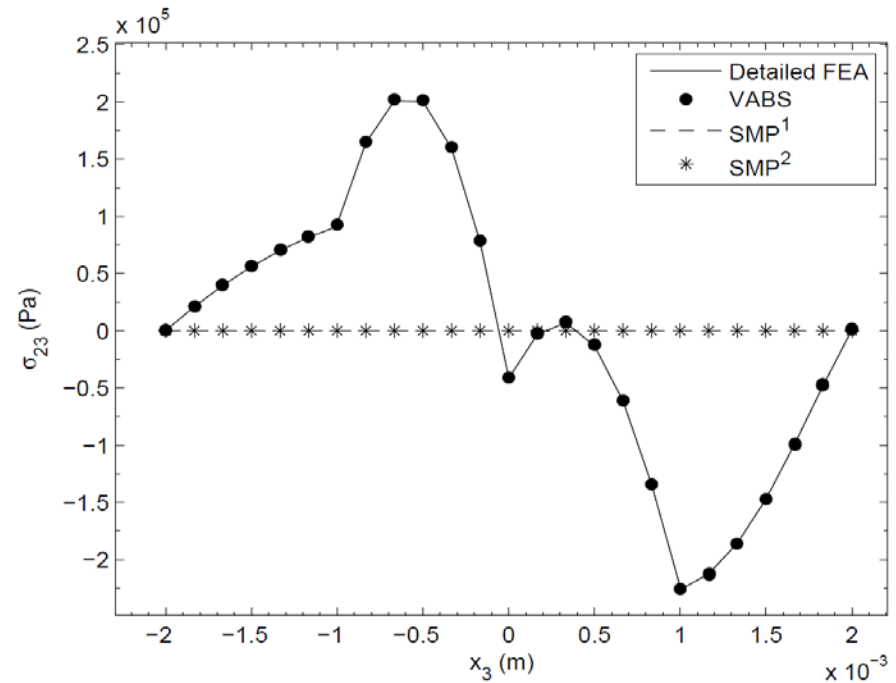
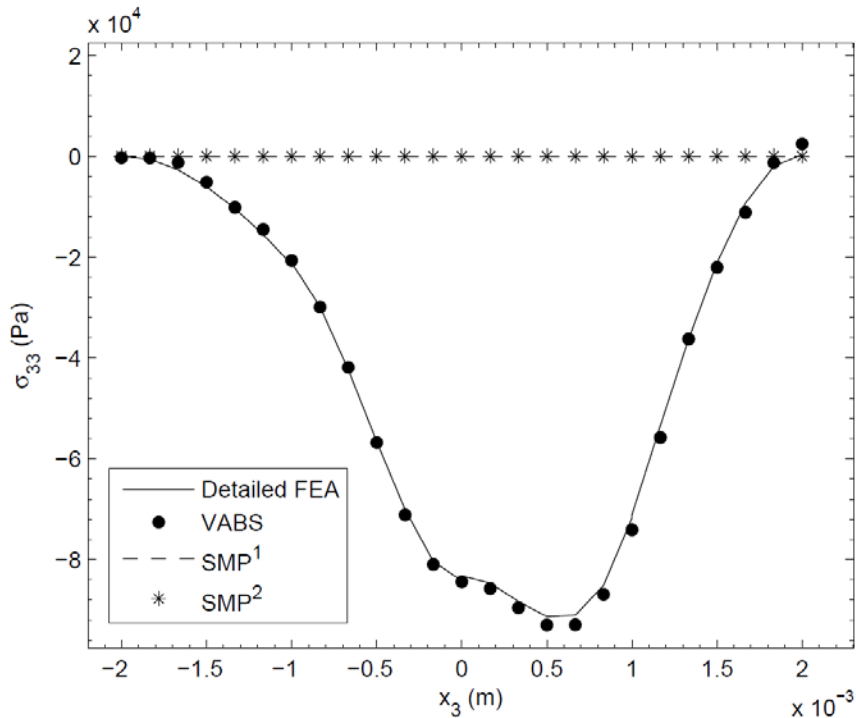
# Loss of Accuracy Using Smeared Properties



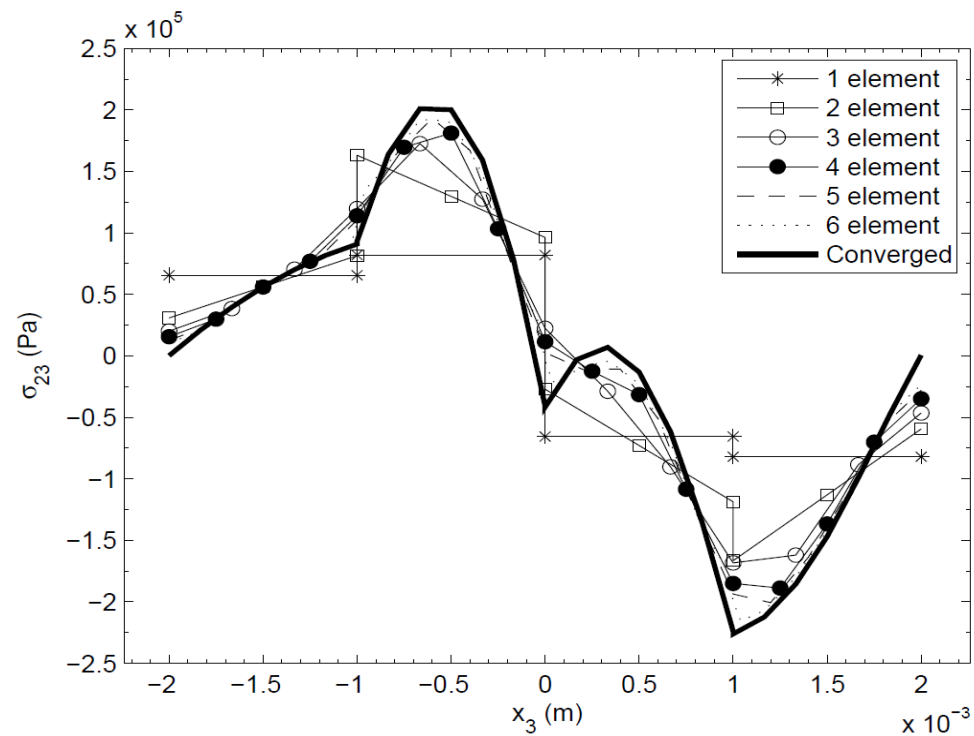
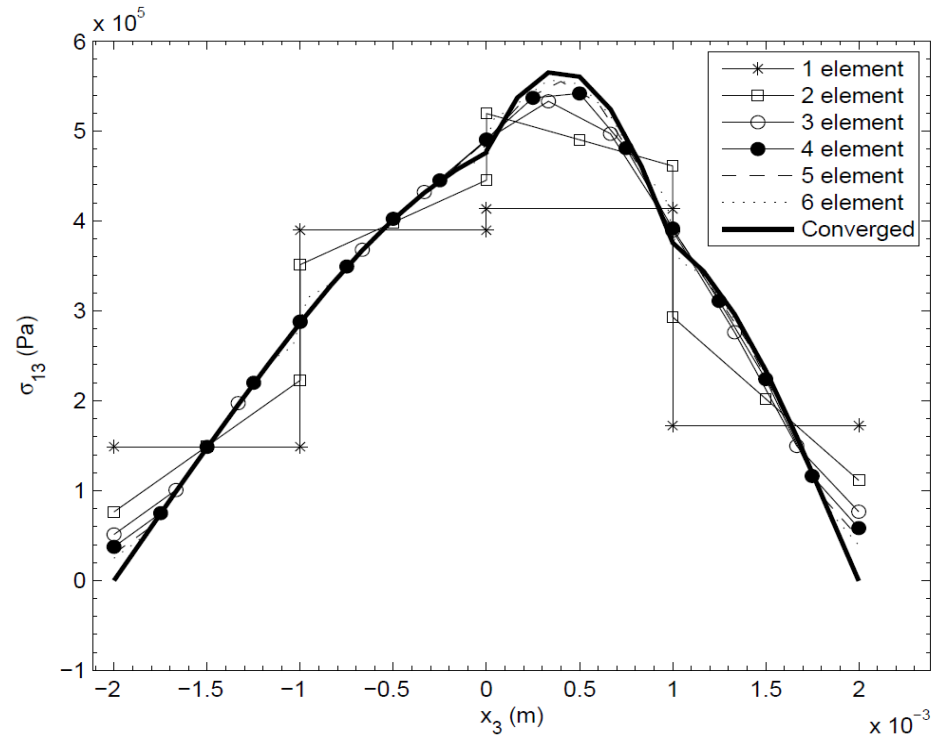
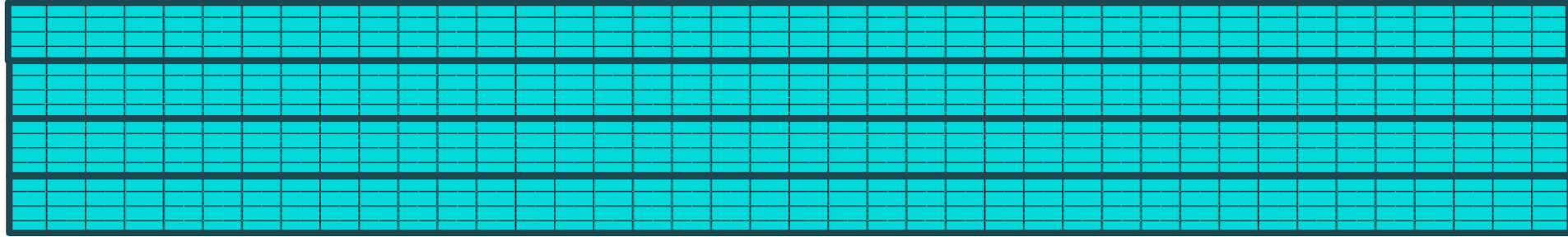
# Loss of Accuracy Using Smeared Properties



# Loss of Accuracy Using Smeared Properties



# Elements/Layer Needed for Direct Numerical Simulation



Converged: six 20-noded brick elements/layer thickness.

# Conclusion

- VABS: an efficient high-fidelity alternative of DNS for slender composite structures.
  - **Best complete set of beam properties:** needed for static/dynamic analysis using beam elements.
  - **Complete set of accurate 3D fields.**
  - **Highly optimized for efficiency:** ply-level details of real blades can be modeled in seconds
  - **Extensively validated** in helicopter and wind industry
  - **Directly integrated** into other design environments
  - **An essential piece for multiscale simulation** of slender composite structures to link properties predicted by micromechanics to structural analysis.