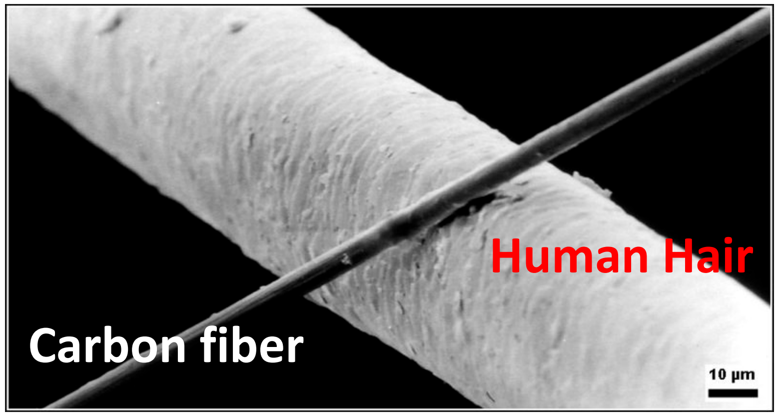
The Best 3D Properties of Composite Laminates

Orzuri Rique, Johnathan Goodsell, Wenbin Yu, & R. Byron Pipes

The Challenge: Multiple Scales



1 mm^3 material block

~ **20 Million DOFs**

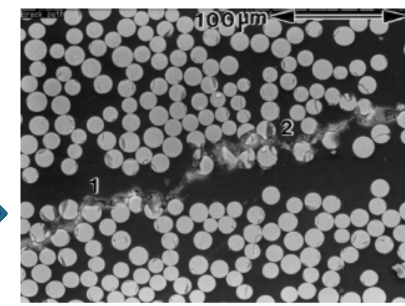
Top-Down Multiscale Modeling

Structural Analysis **Microstructure**

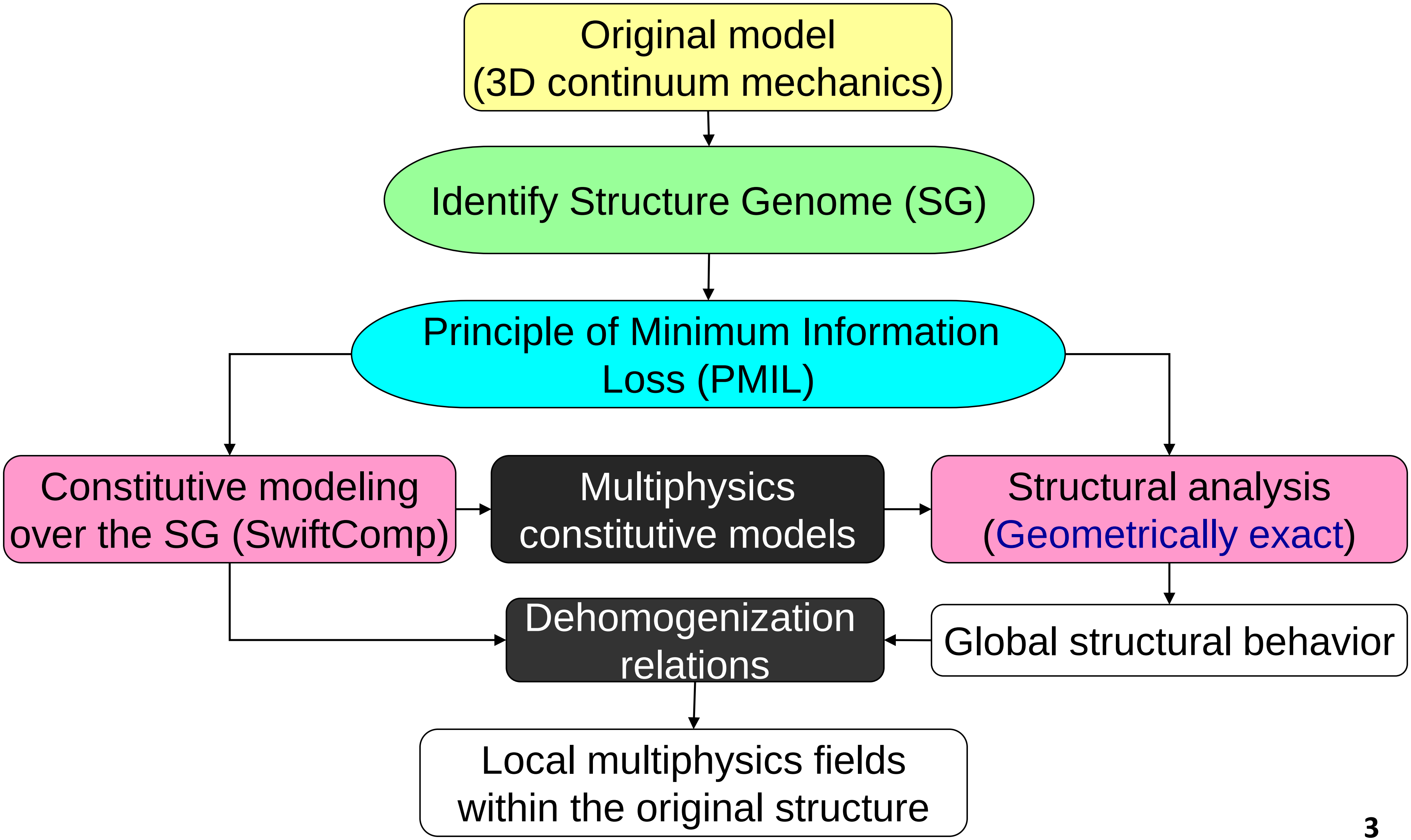
10^1 m 10^{-6} m

Mechanics of Structure Genome

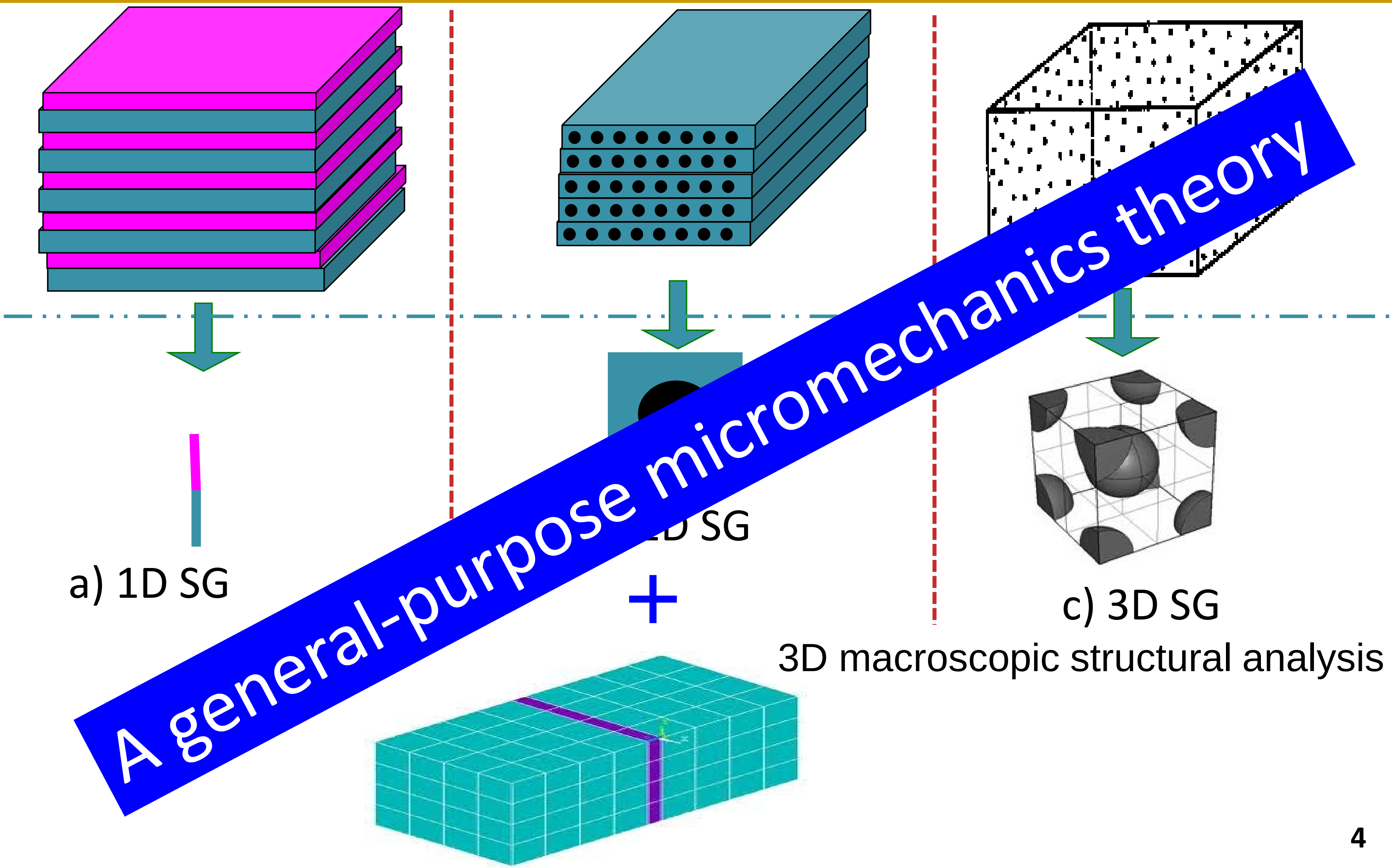
Minimize Information Loss



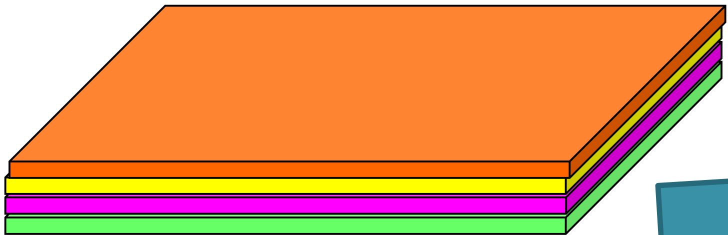
Mechanics of Structure Genome



SG for 3D Structures

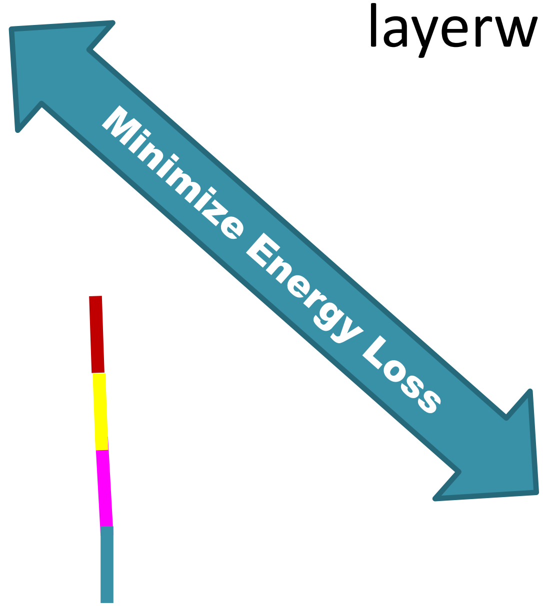


3D Properties of Composite Laminates



Original model: 3D continuum mechanics with layerwise heterogeneity

$$u_i, \varepsilon_{ij}, \sigma_{ij}, U$$



Macroscopic model: 3D continuum mechanics with homogenous solid

$$\bar{u}_i, \bar{\varepsilon}_{ij}, \bar{\sigma}_{ij}, \bar{U}$$

3D Properties of Composite Laminates (cont.)

- Express kinematics of the original model in terms of that of the macroscopic model and unknown fluctuating functions

$$u_i = \bar{u}_i + w_i$$

$$\varepsilon_{ij} = \bar{\varepsilon}_{ij} + \frac{1}{2}(w_{i,j} + w_{j,i})$$

- Define kinematics of the macroscopic model in terms of the original model

3D Properties of Composite Laminates (cont.)

- Express the energy of the original model

$$U = U(\varepsilon_{ij}) = U(\bar{\varepsilon}_{ij}, w_{i,j})$$

- Minimize the energy to solve fluctuating functions

$$\min_{w_i} U(\bar{\varepsilon}_{ij}, w_{i,j}) = \bar{U}(\bar{\varepsilon}_{ij})$$

- **Result: in-plane strains are constant and transverse stresses are constant**

MSG-Based Hybrid Rule of Mixtures for Composite Laminates

$$\begin{Bmatrix} \sigma_e \\ \sigma_t \end{Bmatrix} = \begin{bmatrix} C_e & C_{et} \\ C_{et}^T & C_t \end{bmatrix} \begin{Bmatrix} \varepsilon_e - \alpha_e \Delta T \\ \varepsilon_t - \alpha_t \Delta T \end{Bmatrix}$$

$$\begin{Bmatrix} \sigma_e \\ \varepsilon_t - \alpha_t \Delta T \end{Bmatrix} = \begin{bmatrix} Q & C_{et} C_t^{-1} \\ -C_t^{-1} C_{et}^T & C_t^{-1} \end{bmatrix} \begin{Bmatrix} \varepsilon_e - \alpha_e \Delta T \\ \sigma_t \end{Bmatrix}$$

$$\begin{aligned} \overline{\sigma}_e = \langle \sigma_e \rangle &= \langle Q \rangle \overline{\varepsilon}_e - \langle Q \alpha_e \rangle \Delta T + \langle C_{et} C_t^{-1} \rangle \overline{\sigma}_t \\ &= Q^* (\overline{\varepsilon}_e - \alpha_e^* \Delta T) + C_{et}^* (C_t^*)^{-1} \overline{\sigma_t} \end{aligned}$$

$$\langle \varepsilon_t - \alpha_t \Delta T \rangle = \left\langle -C_t^{-1} C_{et}^T \right\rangle \bar{\varepsilon}_e + \left\langle C_t^{-1} C_{et}^T \alpha_e \right\rangle \Delta T + \left\langle C_t^{-1} \right\rangle \bar{\sigma}_t$$

$$\begin{Bmatrix} \overline{\sigma}_e \\ \overline{\sigma}_t \end{Bmatrix} = \begin{bmatrix} C_e^* & C_{et}^* \\ {C_{et}^*}^T & C_t^* \end{bmatrix} \begin{Bmatrix} \overline{\varepsilon}_e - \alpha_e^* \Delta T \\ \overline{\varepsilon}_t - \alpha_t^* \Delta T \end{Bmatrix}$$

MSG-Based Hybrid Rule of Mixtures for Composite Laminates

$$C_t^* = \langle C_t^{-1} \rangle^{-1}$$

$$C_{et}^* = \langle C_{et} C_t^{-1} \rangle C_t^*$$

$$C_e^* = Q^* + C_{et}^* {C_t^*}^{-1} C_{et}^* T$$

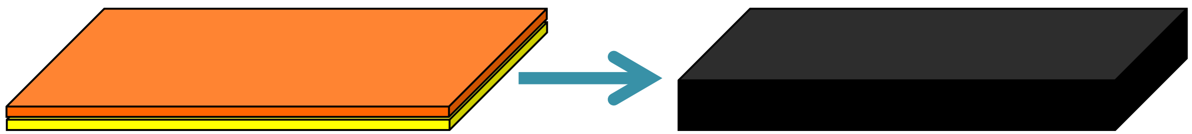
$$\alpha_e^* = (Q^*)^{-1} \langle Q \alpha_e \rangle$$

$$\alpha_t^* = \langle \alpha_t + C_t^{-1} C_{et}^T \alpha_e \rangle - {C_t^*}^{-1} {C_{et}^*}^T \alpha_e^*$$

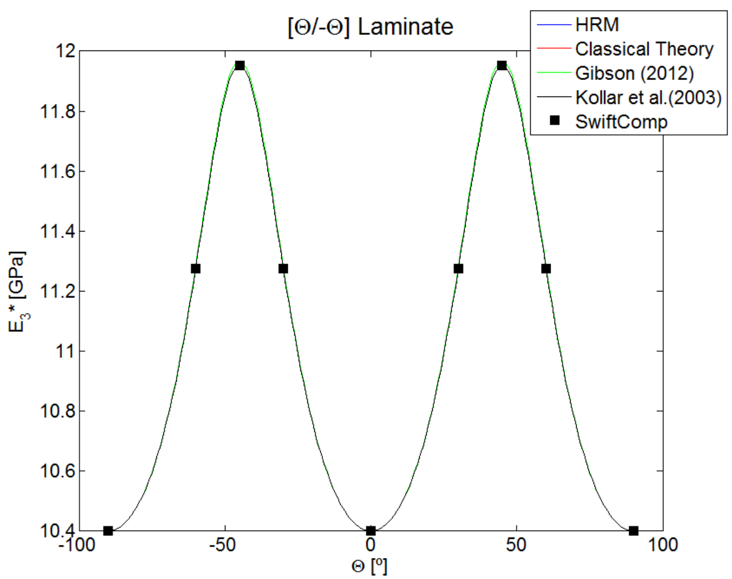
Best complete set of effective thermoelastic properties of composite laminates

Comparison with Other Theories

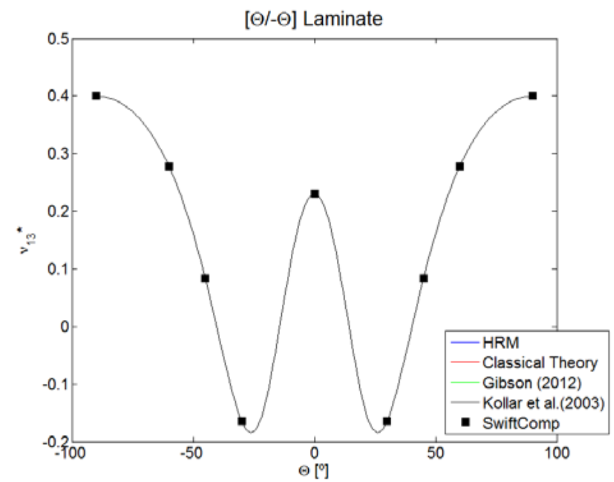
[θ/-θ] Laminate

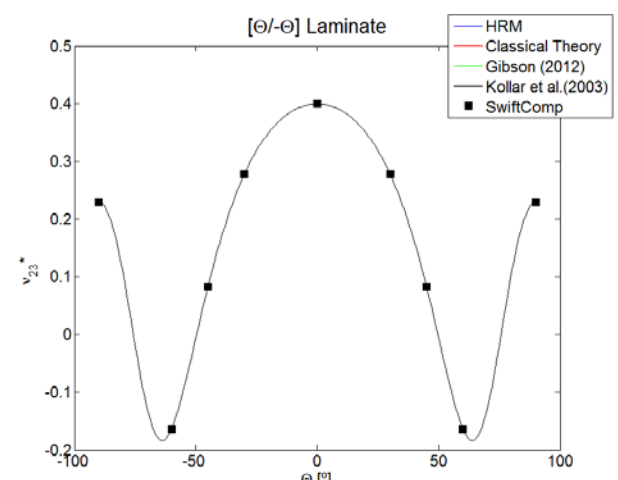


Homogenized transverse Young's modulus E^*_3



Homogenized transverse Poisson's ratios, ν^*_{13} & ν^*_{23}

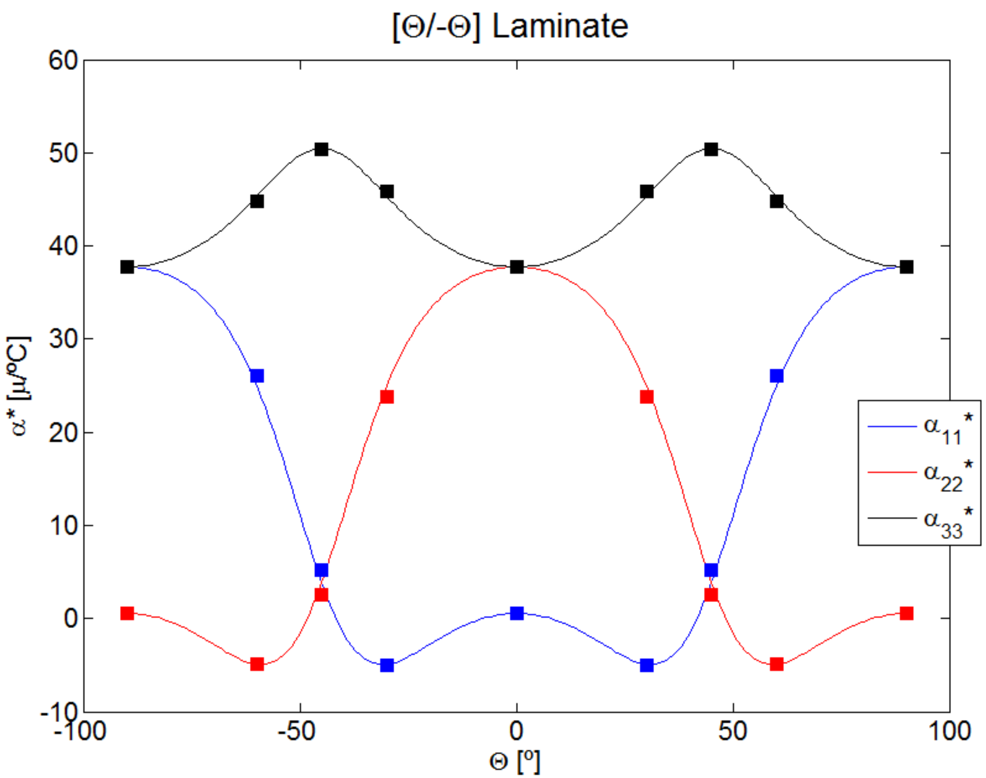




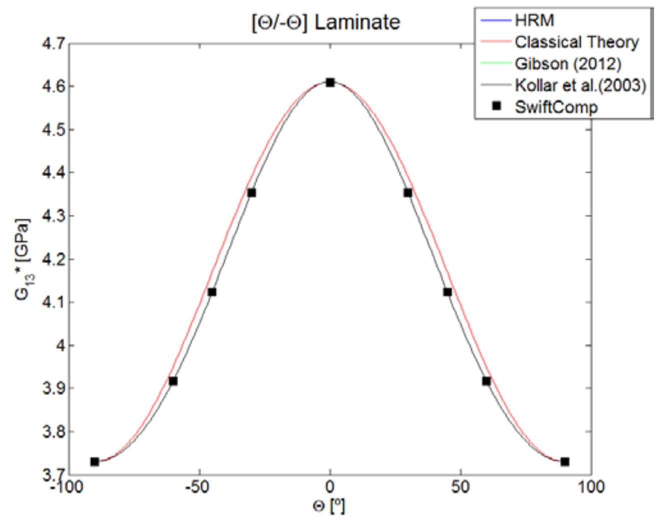
- Hashin Z., Rosen B.W. and Pipes R.B. (1979). *Nonlinear Effects on Composite Laminate Thermal Expansion*, Blue Bell, PA: NASA Contractor Report 3038.
- Gibson A.G. (2012). Through-thickness elastic constants of composite laminates. *Journal of Composite Materials*, 47(28), pp. 3487-3499.
- Kollar L.P. and Springer G.S. (2003). *Mechanics of Composite Structures*, 1st edn. New York, NY: Cambridge University Press

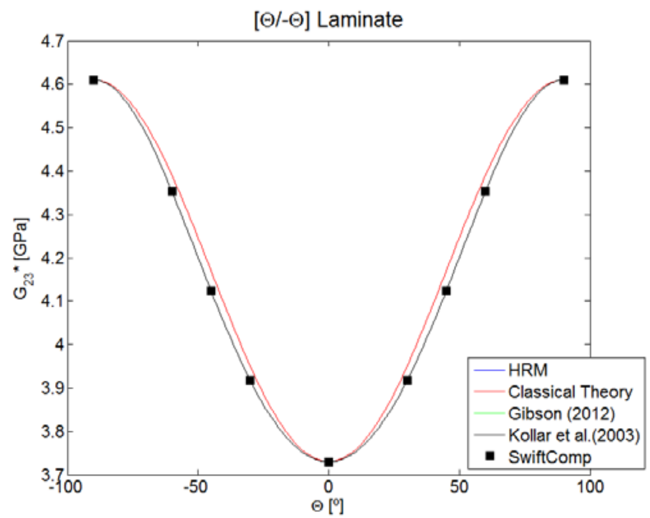
Comparison with Other Theories

Homogenized Coefficients of Thermal Expansion, α^*_1, α^*_2 & α^*_3



Homogenized transverse shear modulus G^*_{13} & G^*_{23}





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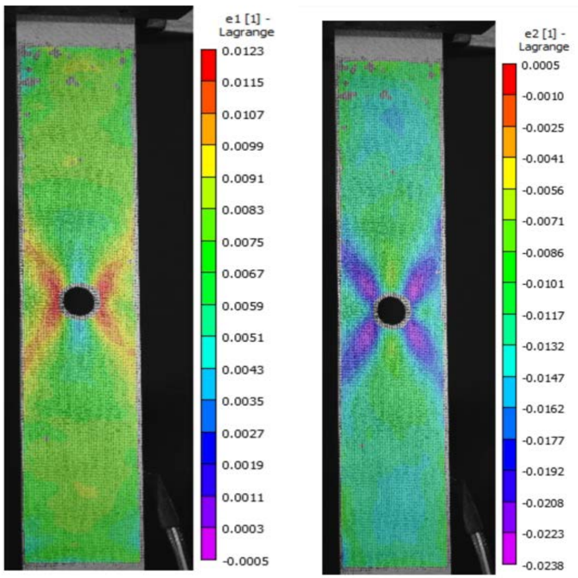
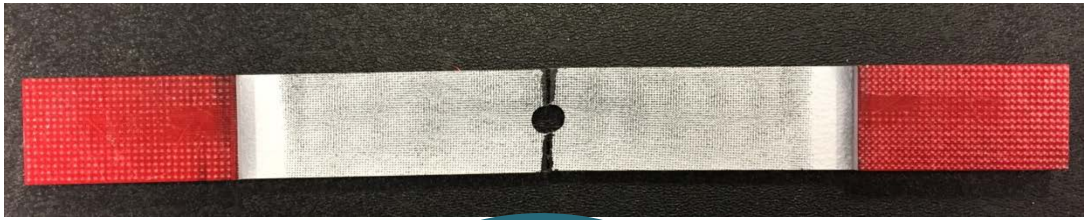
Open Hole – Case Definition

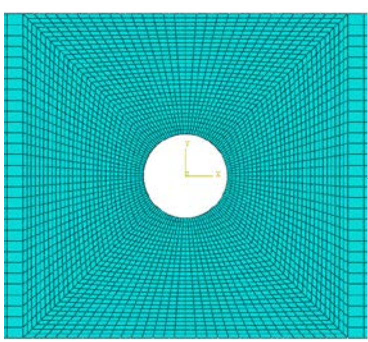
Digital Image Correlation (DIC) - Strains at Failure

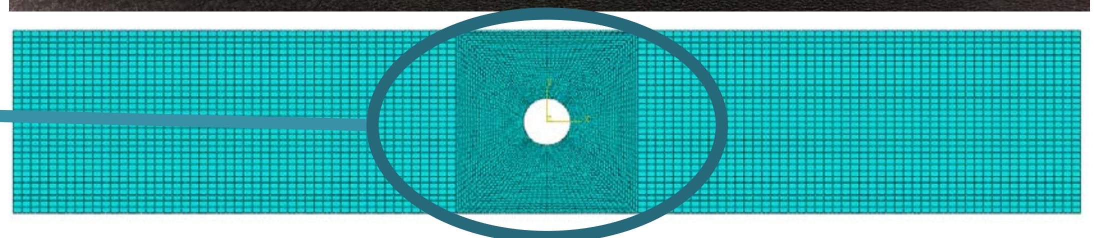


ε_{11} ε_{22}

Actual $[\pm 25]_{2S}$ laminate		Homogenized Laminate -HRM	
Lamina Engineering Constants	Values	Laminate Engineering Constants	Values
$E_1\,[GPa]$	140.00	E_1 [GPa]	65.68
$E_2 = E_3[GPa]$	8.40	E_2 [GPa]	8.968
$\upsilon_{12} = \upsilon_{13}$	0.28	E_3 [GPa]	9.504
υ_{23}	0.55	G_{12} [GPa]	22.99
$G_{12} = G_{13}[GPa]$	4.37	G_{13} [GPa]	3.939
$G_{23}\,[GPa]$	2.71	G_{23} [GPa]	2.907
		υ_{12}	1.561
		υ_{13}	-0.3785
		υ_{23}	0.4381

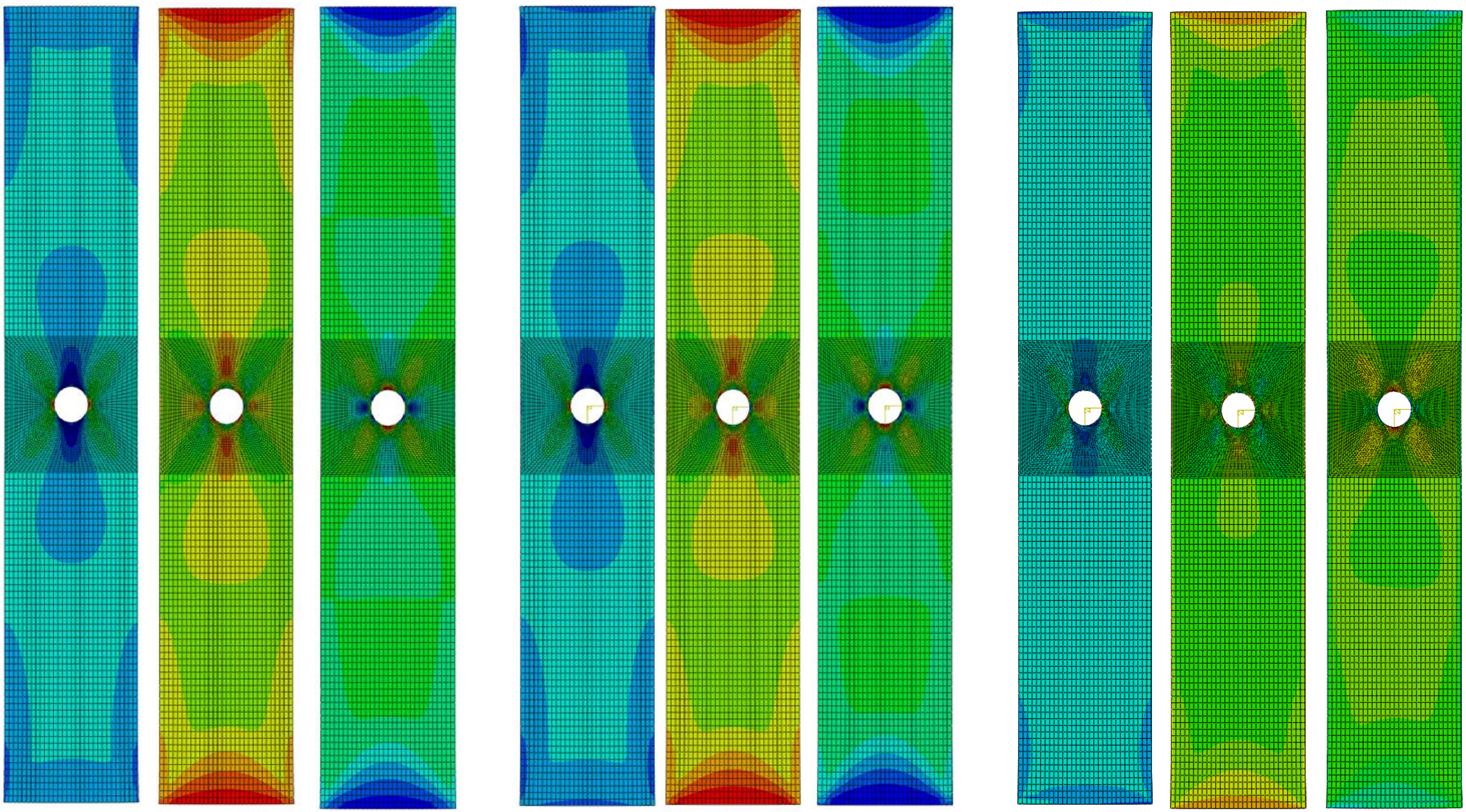






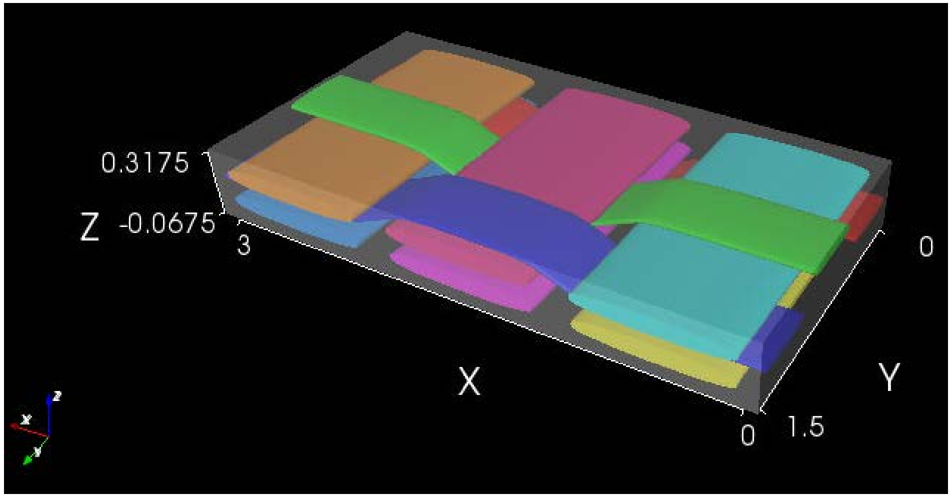
Open Hole – Results

Hybrid Rule of Mixtures | ***Composite Section - Abaqus*** | ***DNS - Abaqus***



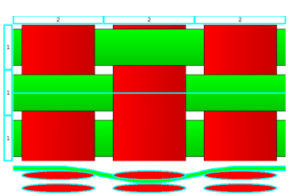
ε_{11} ε_{22} ε_{33} | ε_{11} ε_{22} ε_{33} | ε_{11} ε_{22} ε_{33}

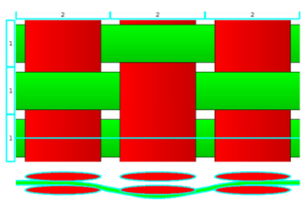
Woven Composite Laminate

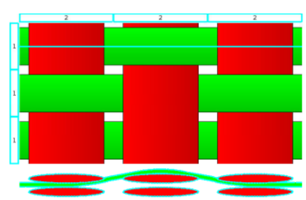


TexGen4SC - cdmHUB

Yarn Properties		Matrix Properties	
Engineering Constants	Values	Engineering Constants	Values
E_1 [GPa]	200	E_m [GPa]	3
$E_2 = E_3$ [GPa]	10	υ_m	0.20
$G_{12} = G_{13} = G_{23}$ [GPa]	5		
υ_{12}	0.30		
$\upsilon_{13} = \upsilon_{23}$	0.40		


Upper


Middle


Lower

Theory	Gibson	Kollar	Classical	HRM
Isotropic	✓	✓	✓	✓
Trans. Iso.	✓	✓	✓	✓
Orthotropic	✓	✓	✓	✓
Monoclinic		✓	✓	✓
Anistropic			✓	✓

$$C = \begin{pmatrix} 1.18227\times10^{10} & 1.5145\times10^{9} & 1.4095\times10^{9} & -450827. & 252456. & -193012. \\ 1.5145\times10^{9} & 5.68331\times10^{10} & 1.65404\times10^{9} & 310152. & -39561.2 & 21653.5 \\ 1.4095\times10^{9} & 1.65404\times10^{9} & 4.78868\times10^{9} & -155005. & -34434.8 & 7194.56 \\ -450827. & 310152. & -155005. & 1.87857\times10^{9} & -7989.69 & 12382.4 \\ 252456. & -39561.2 & -34434.8 & -7989.69 & 1.81113\times10^{9} & 441376. \\ -193012. & 21653.5 & 7194.56 & 12382.4 & 441376. & 2.13641\times10^{9} \end{pmatrix}$$

Fully Populated Effective Stiffness Matrix

Woven Composite Laminate – $[0/90]_s$ Laminate

Lamina Engineering Constants – Approximated as Orthotropic	
Laminate Engineering Constants	**Values**
E_1 [GPa]	11.3890
E_2 [GPa]	56.1692
E_3 [GPa]	4.5823
G_{12} [GPa]	2.1364
G_{13} [GPa]	1.8111
G_{23} [GPa]	1.8786
υ_{12}	0.0183
υ_{13}	0.2880
υ_{23}	0.3189

Laminate Engineering Constants – Approximated as Orthotropic				
Laminate Engineering Constants	**Gibson (2012)**	**Kollar et al (2003)**	**Classical Theory**	**HRM**
E_1 [GPa]	33.8069*	33.8000	33.8069	33.8036
E_2 [GPa]	33.8069*	33.800	33.8069	33.8036
E_3 [GPa]	4.6575	4.6800	4.6578	4.6578
G_{12} [GPa]	2.1361*	2.1361	2.1361	2.1364
G_{13} [GPa]	1.8789	1.8440	1.8449	1.8442
G_{23} [GPa]	1.8111	1.8440	1.8449	1.8442
υ_{12}	0.0303*	0.0304	0.0303	0.0304
υ_{13}	0.3365	0.3102	0.3102	0.3102
υ_{23}	0.2838	0.3102	0.3102	0.3102

****Values predicted using Classical Theory***

Woven Composite Laminate – $[0/90]_s$ Laminate

Classical Theory (Full Anisotropy) – Effective Stiffness Matrix for $[0/90]_s$

$$\begin{pmatrix} 3.43248\times10^{10} & 1.51763\times10^{9} & 1.53177\times10^{9} & -242929. & -26097.4 & -107156. \\ 1.51763\times10^{9} & 3.43248\times10^{10} & 1.53177\times10^{9} & 282271. & 203541. & 107174. \\ 1.53177\times10^{9} & 1.53177\times10^{9} & 4.78868\times10^{9} & -93618.3 & 58553.5 & 18.4268 \\ -242929. & 282271. & -93618.3 & 1.84423\times10^{9} & -1.14472 & -218643. \\ -26097.4 & 203541. & 58553.5 & -1.14472 & 1.84423\times10^{9} & 230801. \\ -107156. & 107174. & 18.4268 & -218643. & 230801. & 2.13641\times10^{9} \end{pmatrix}$$

HRM (Full Anisotropy) – Effective Stiffness Matrix for $[0/90]_s$

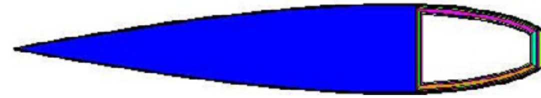
$$\begin{pmatrix} 3.43248\times10^{10} & 1.51763\times10^{9} & 1.53177\times10^{9} & -242929. & -26097.4 & -107156. \\ 1.51763\times10^{9} & 3.43248\times10^{10} & 1.53177\times10^{9} & 282271. & 203541. & 107174. \\ 1.53177\times10^{9} & 1.53177\times10^{9} & 4.78868\times10^{9} & -93618.3 & 58553.5 & 18.4268 \\ -242929. & 282271. & -93618.3 & 1.84423\times10^{9} & -1.14472 & -218643. \\ -26097.4 & 203541. & 58553.5 & -1.14472 & 1.84423\times10^{9} & 230801. \\ -107156. & 107174. & 18.4268 & -218643. & 230801. & 2.13641\times10^{9} \end{pmatrix}$$

Kollar (Monoclinic) – Effective Stiffness Matrix for $[0/90]_s$

$$\begin{pmatrix} 3.43248\times10^{10} & 1.51763\times10^{9} & 1.53177\times10^{9} & 0 & 0 & -107156. \\ 1.51763\times10^{9} & 3.43248\times10^{10} & 1.53177\times10^{9} & 0 & 0 & 107174. \\ 1.53177\times10^{9} & 1.53177\times10^{9} & 4.78868\times10^{9} & 0 & 0 & 18.4268 \\ 0 & 0 & 0 & 1.84423\times10^{9} & -1.14472 & 0 \\ 0 & 0 & 0 & -1.14472 & 1.84423\times10^{9} & 0 \\ -107156. & 107174. & 18.4268 & 0 & 0 & 2.13641\times10^{9} \end{pmatrix}$$

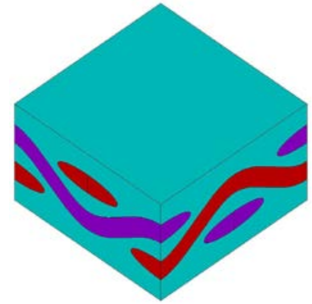
Conclusion

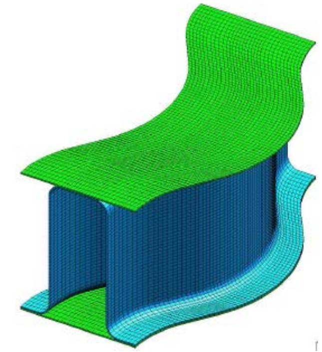
- MSG is applied to homogenize composite laminates.
- A simple hybrid rule of mixture is developed.
- Provides complete set of thermoelastic properties for laminates with general layup and general anisotropic materials.
- MSG-based hybrid rule of mixtures have a good agreement with existing approaches for orthotropic materials.

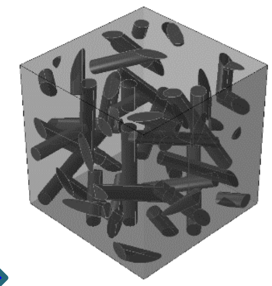


Right Results

Right Away



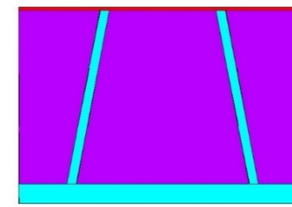


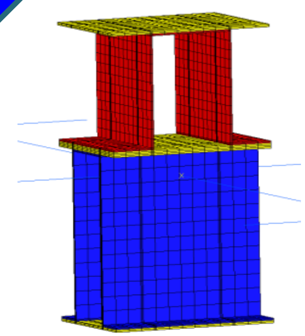


SwiftComp™

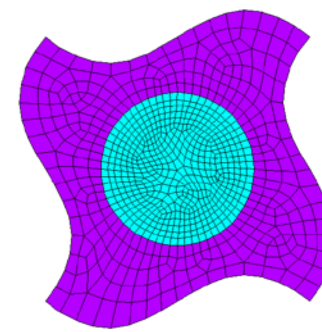
A Purdue Technology

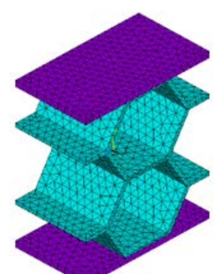
Principle of Minimum Information Loss





- **Virtual testing of materials**
  - Mechanical properties
  - Multifunctional properties





- **Multiscale modeling of structures**
  - Composite laminates
  - Build-up structures: stiffened, sandwiched, corrugated