Calculation of stress relaxation stiffness of linear viscoelastic composites using ABAQUS

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Introduction

- This tutorial demonstrates a method of calculating the effective stress relaxation stiffness of linear viscoelastic composites using ABAQUS.
- A fiber reinforced polymer matrix composite is used as an example in which the polymer is assumed to be *linear* viscoelastic behavior while the fiber is linear elastic material.
- The present methodology can be readily extended to other types of composites such as particle reinforced composites, chopped fiber reinforced composites, etc.

Periodic boundary conditions

The composites can be idealized as assembly of many periodic unit cells to which the periodic boundary conditions are consequently applied, which means that the deformation mode in each unit cell are identical and there is no gap or overlap between the adjacent unit cells. The periodic boundary conditions are represented as

$$u_i = \bar{\varepsilon}_{ij} x_j + v_i \tag{1}$$

where $\bar{\varepsilon}_{ij}$ is the average strain; v_i is the periodic part of the displacement components also called local fluctuation on the boundary surfaces. The displacements on a pair of opposite boundary surfaces are given by

$$u_i^{k+} = \bar{\varepsilon}_{ij} x_j^{k+} + v_i^{k+} \tag{2}$$

$$u_i^{k-} = \bar{\varepsilon}_{ij} x_j^{k-} + v_i^{k-} \tag{3}$$

where "k +" denotes along the positive x_j direction while "k –" means along the negative x_j direction. Since the periodic parts v_i^{k+} and v_i^{k-} are identical on the two opposite boundary surfaces of a periodic unit cell, the difference of Eq. (2) and (3) is obtained as

$$u_i^{k+} - u_i^{k-} = \bar{\varepsilon}_{ij} \left(x_j^{k+} - x_j^{k-} \right) = \bar{\varepsilon}_{ij} \Delta x_j \tag{4}$$

where Δx_j is actually the edge length of the unit cell.

- The periodic boundary conditions described in Eq. (4) are implemented into a python script.
- The periodic boundary conditions described in Eq. (4) were applied to the unit cell by coupling opposite nodes on corresponding opposite boundary surfaces. In actual manipulation, three reference points are first created and their displacements are assigned as $\bar{\varepsilon}_{ij}\Delta x_j$.

Unit cell of a fiber reinforced composite



- In this study, the fiber is assumed to be of circular shape and in square array.
- The unit cell model is meshed by C3D8R elements.
- Sweep mesh technique was used in order to obtain periodic mesh on opposite boundary surfaces, which means that the meshes on opposite boundary surfaces are identical.
- > In the present study, the fiber direction is along -1. The edge length of the unit cell along 1, 2, and 3 direction are respectively $\Delta x_1 = 0.1$ mm and $\Delta x_2 = \Delta x_3 = 1$ mm.

Material properties of the constituents

In the present example, the **ABAQUS** unit cell model was used to calculate the effective stress relaxation stiffness and creep compliance of the glass fiber reinforced polymer matrix composites.

- > The glass fibers are isotropic and linear elastic materials with Young's modulus and Poisson's ratio being 80,000 MPa and 0.3 respectively. The volume fraction of the fibers is vof = 20%.
- The elastic relaxation modulus of the isotropic and linear viscoelastic polymer materials can be expressed using Prony series in the following way:

$$E(t) = E_0 \left(1 - \sum_{k=1}^n g_k (1 - e^{-t/\tau_k}) \right) = E_\infty + \sum_{k=1}^n E_i e^{-t/\tau_k}$$

where E_0 is the instantaneous Young's modulus and also given by

 $E_0 = E_{\infty} + \sum_{k=1}^n E_i = E_{\infty} + \sum_{k=1}^n E_0 g_k$ with E_{∞} being the long-term Young's modulus;

 τ_k is the time relaxation material parameter.

Material properties of the constituents (cont.)

Table 1. Relaxation times and Prony coefficients for PMT-F4 epoxy.*

i	E_i , MPa	ρ_i , s
00	1000	
1	224.1	1.0e + 3
2	450.8	1.0e + 5
3	406.1	1.0e + 6
4	392.7	1.0e + 7
5	810.4	1.0e + 8
6	203.7	1.0e + 9
7	1486.0	1.0e + 10

* Kawai Kwok and Sergio Pellegrino, "Micromechanics models for viscoelastic plain-weave composite tape springs", AIAA Journal, Vol. 55, No. 1, January 2017.

In the present example, the material properties of the polymer matrix is shown in left table. The Poisson's ratio of the polymer is 0.33.

$$E_0 = E_{\infty} + \sum_{i=1}^7 E_i = 4973.8 \text{ Mpa}$$

$$g_1 = \frac{E_1}{E_0} = 0.045056094; \quad g_2 = \frac{E_2}{E_0} = 0.090634927$$

$$g_3 = \frac{E_3}{E_0} = 0.081647835; \quad g_4 = \frac{E_4}{E_0} = 0.0789553717$$

$$g_5 = \frac{E_5}{E_0} = 0.162933773; \quad g_6 = \frac{E_6}{E_0} = 0.040954602$$

$$g_7 = \frac{E_7}{E_0} = 0.045056094$$
where g_i is the dimensionless Young's modulus.

Material inputs of the polymer in ABAQUS unit cell model

🖶 Edit Material	🖶 Edit Material	×
Name: resinKawaiKwok-Instaneous	Name: resinKawaiKwok-Instaneous	
Description:	Description:	
Material Behaviors	Material Behaviors	
Elastic	Elastic	
Viscoelastic	Viscoelastic	
<u>G</u> eneral <u>M</u> echanical <u>I</u> hermal <u>E</u> lectrical/Magnetic <u>O</u> ther	<u>G</u> eneral <u>M</u> echanical <u>T</u> hermal <u>E</u> lectrical/Magnetic <u>O</u> ther	
Elastic	Viscoelastic	
Type: Isotropic Suboptions	Domain: Time	▼ Test Data
🔲 Use temperature-dependent data	Time: Prony	▼ Suboptions
Number of field variables: 0	Type: Sotropic Traction	
Moduli time scale (for viscoelasticity): Instantaneous 💌	Preload: 🔘 None 🔘 Uniaxial 🔘 Volumetric 🔘 Uniaxial and Volumetric	
No compression	Maximum number of terms in the Prony series: 13 🛋	
No tension	Allowable average root-mean-square error: 0.01	
Data	Data	
Young's Poisson's Modulus Batio	g_i Prony k_i Prony tau_i Prony	
1 4973.8 0.33	1 0.045056094 0.045056094 1000	
	2 0.090634927 0.090634927 100000	
	4 0.078953717 0.078953717 1000000	
	5 0.162933773 0.162933773 10000000	
	6 0.040954602 0.040954602 100000000	
	7 0.298765531 0.298765531 1000000000	
OK	ΟΚ	Cancel

Note that $g_i = k_i$ with k_i being the dimensionless bulk modulus.

Effective stress relaxation stiffness

The effective properties of fiber reinforced composites with the fibers in square array possess square symmetry. The effective stress relaxation stiffness matrix can be expressed as:

$(\bar{\sigma}_{11}(t))$		$C_{11}^{*}(t)$	$C_{12}^{*}(t)$	$C_{12}^{*}(t)$	0	0	0 -	$(\bar{\epsilon}_{11}^{cst})$	
$\bar{\sigma}_{22}(t)$		$C^*_{12}(t)$	$C^*_{22}(t)$	C_{23}^{*}	0	0	0	$\bar{\epsilon}_{22}^{cst}$	
$\bar{\sigma}_{33}(t)$	_	$C_{12}^{*}(t)$	$C^*_{23}(t)$	C_{22}^{*}	0	0	0	$\overline{\mathcal{E}}_{33}^{cst}$	
$\bar{\sigma}_{23}(t)$	-	0	0	0	$C^*_{44}(t)$	0	0	$\bar{\gamma}_{23}^{cst}$	
$\bar{\sigma}_{12}(t)$		0	0	0	0	$C_{55}^{*}(t)$	0	$\bar{\gamma}_{12}^{cst}$	
$\sigma_{13}(t)$		0	0	0	0	0	$C_{55}^{*}(t)$	$\bar{\gamma}_{13}^{cst}$	

Where "cst" means constant values that do not vary with time but may change with position.

Load cases

In order to calculate the full set of stress relaxation stiffness of the linear viscoelastic composites, four load cases are applied:

- > Load case 1: The constant macroscopic strain $\bar{\varepsilon}_{11} = 0.1$ along 1 direction was applied by prescribing the 1 direction displacement of Reference point-1 as $u_1 = 0.01$. All other mechanical strains are set to zero.
- > Load case 2: The constant macroscopic strain $\bar{\varepsilon}_{22} = 0.1$ along 2 direction was applied by prescribing the 2 direction displacement of Reference point-2 as $u_2 = 0.1$. All other mechanical strains are set to zero.

Reference point-2

- > Load case 3: The constant macroscopic transverse shear strain $\bar{\gamma}_{23} = 0.1$ was applied by prescribing the 3 direction displacement of Reference point-2 as $u_3 = 0.1$ and 2 direction displacement of Reference point-3 as $u_2 = 0.1$. All other mechanical strains are set to zero.
- Load case 4: The constant macroscopic longitudinal shear strain $\bar{\gamma}_{12} = 0.1$ was applied by prescribing the 2 direction displacement of Reference point-1 as $u_2 = 0.01$ and 1 direction displacement of Reference point-2 as $u_1 = 0.1$. All other mechanical strains are set to zero.

Stress relaxation loading



The constant strain was applied since t = 0.

Calculation of $C_{11}^{*}(t)$ and $C_{12}^{*}(t)$ The calculation of $C_{11}^*(t)$ and $C_{12}^*(t)$ are calculated under **Load case 1**. Contour plot of von Mises stress of unit cell [×1.E7 under Load case 1. 📛 History Output Variables Steps/Frames Output Variables Name filter: X History Output Variables Steps/Frame Reaction force: RE1 PI: REEPOINT-2-1 Node 1 in NSET SETREEPOINT Output Variable Reaction force: RF1 PI: REEPOINT-3-1 Node 1 in NSET SE Name filter: Reaction force: RF2 PI: REFPOINT-1-1 Node 1 in NSET SETREFPOINT1 Reaction force: RE2 PI: REEPOINT-2-1 Node 1 in NSET SETREEPOINT2 Reaction force: RE1 PI: REEPOINT-1-1 Node 1 in NSET SETREEPOINT Reaction force: RE2 PI: REEPOINT-3-1 Node 1 in NSET SETREEPOINT Reaction force: RE3 PI: REEPOINT-1-1 Node 1 in NSET SETREEPOINT Reaction force: RF3 PI: REFPOINT-2-1 Node 1 in NSET SETREFPOINT2 Reaction force: RE3 PI: REEPOINT-3-1 Node 1 in NSET SETREEPOINT3 Total time in which creep was active: CRPTIME for Whole Mo Reaction force: RE3 DF REEDOINT-2-1 Node 1 in NSE Reaction force: RE3 DI: REEDOINT_3_1 Node 1 in NSET SETREEDOINT Save As... Plot Dismiss Total time in which creep was active: CRPTIME for Whole M Plot Save As... Dismiss The $C_{11}^*(t)$ was calculated as $C_{11}^{*}(t) = RF_1(t)/(A_1\bar{\varepsilon}_{11})$ where $RF_1(t)$ is the variation of 1 component The $C_{12}^{*}(t)$ was calculated as of the reaction force of Reference point-1 $C_{12}^{*}(t) = RF_2(t)/(A_2\bar{\varepsilon}_{11})$ where $RF_2(t)$ is the variation of 2 component of the reaction force of Reference point-2

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Calculation of $C_{22}^{*}(t)$ and $C_{23}^{*}(t)$

The calculation of $C_{22}^{*}(t)$ and $C_{23}^{*}(t)$ are calculated under **Load case 2**.

	,		Contour plot of von
100 Image: Steps/Frames 0 Utput Variables Output Variables Name filter: Image: Steps/Frames 0 Utput Variables Reaction force: RFJ PE: REFPOINT-1:1 Node1 in NSET SETREFPO Reaction force: RFJ PE: REFPOINT-1:1 Node1 in NSET SETREFPO Reaction force: RFJ PE: REFPOINT-1:1 Node1 in NSET SETREFPO Reaction force: RFJ PE: REFPOINT-1:1 Node1 in NSET SETREFPO Reaction force: RFJ PE: REFPOINT-1:1 Node1 in NSET SETREFPO Reaction force: RFJ PE: REFPOINT-1:1 Node1 in NSET SETREFPO Reaction force: RFJ PE: REFPOINT-1:1 Node1 in NSET SETREFPO Reaction force: RFJ PE: REFPOINT-1:1 Node1 in NSET SETREFPO Reaction force: RFJ PE: REFPOINT-1:1 Node1 in NSET SETREFPO Reaction force: RFJ PE: REFPOINT-1:1 Node1 in NSET SETREFPO Reaction force: RFJ PE: REFPOINT-1:1 Node1 in NSET SETREFPO Reaction force: RFJ PE: REFPOINT-1:1 Node1 in NSET SETREFPO Reaction force: RFJ PE: REFPOINT-1:1 Node1 in NSET SETREFPO Reaction force: RFJ PE: REFPOINT-1:1 Node1 in NSET SETREFPO Reaction force: RFJ PE: REFPOINT-1:1 Node1 in NSET SETREFPO Reaction force: RFJ PE: REFPOINT-1:1 Node1 in NSET SETREFPO Reaction force	II III IIII IIII IIII IIIII<	History Output History Output Viriables Mare filter: Name filter: Name filter: Name filter: Reaction force: RF1 PI: REFPOINT-1:1 Node 1 in NSET SET REFPOINT1 Reaction force: RF1 PI: REFPOINT-2:1 Node 1 in NSET SET REFPOINT1 Reaction force: RF1 PI: REFPOINT-1:1 Node 1 in NSET SET REFPOINT1 Reaction force: RF1 PI: REFPOINT-1:1 Node 1 in NSET SET REFPOINT1 Reaction force: RF1 PI: REFPOINT-1:1 Node 1 in NSET SET REFPOINT1 Reaction force: RF2 PI: REFPOINT-1:1 Node 1 in NSET SET REFPOINT1 Reaction force: RF2 PI: REFPOINT-1:1 Node 1 in NSET SET REFPOINT1 Reaction force: RF3 PI: REFPOINT-1:1 Node 1 in NSET SET REFPOINT1 Reaction force: RF3 PI: REFPOINT-1:1 Node 1 in NSET SET REFPOINT1 Reaction force: RF3 PI: REFPOINT-1:1 Node 1 in NSET SET REFPOINT1 Reaction force: RF3 PI: REFPOINT-1:1 Node 1 in NSET SET REFPOINT1 Reaction force: RF3 PI: REFPOINT-1:1 Node 1 in NSET SET REFPOINT1 Reaction force: RF3 PI: REFPOINT-1:1 Node 1 in NSET SET REFPOINT1 Reaction force: RF3 PI: REFPOINT-1:1 Node 1 in NSET SET REFPOINT1 Reaction force: RF3 PI: REFPOINT-1:1 Node 1 in NSET SET REFPOINT1 Reaction force: RF3 PI: REFPOINT-1:1 Node 1 in NSET SET REFPOINT1 Reaction force: RF3 PI: REFPOINT-1:1 Node 1 in NSET SET REFPOINT1 Reaction force: RF3 PI: REFPOINT-1:1 Node 1 in NSET SET REFPOINT1 Reaction force: RF3 PI: REFPOINT-1:1 Node 1 in NSET SET REFPOINT1 Reaction force: RF3 PI: REFPOINT-1:1 Node 1 in NSET SET REFPOINT3 Total time in which creep was active: CR7IME for Whole Model Swe Asin	Mises stress of unit cell under Load case 2.
The $C^*_{22}(t)$ was calculated as $C^*_{22}(t) = RF_2(t)/(A_2\bar{\varepsilon}_{22})$			
where $RF_2(t)$ is the variation of 2 constants of the reaction force of Reference p	omponent oint-2. The C_{23}^* where L_{23}^* of the r	$C_{23}^{*}(t)$ was calculated as $C_{23}^{*}(t) = RF_3(t)/(A_3\bar{\epsilon}_{22})$ $RF_3(t)$ is the variation of 3 components reaction force of Reference point-3	onent 3.

Calculation of $C_{44}^*(t)$

The calculation of $C_{44}^{*}(t)$ is calculated under **Load case 3**.



Contour plot of von Mises stress of unit cell under Load case 3.

Calculation of $C_{55}^{*}(t)$

The calculation of $C_{55}^{*}(t)$ is calculated under **Load case 4**.

50	History Output	
	Variables Steps/Frames Output Variables Name filter: Reaction force: RF1 PI: REFPOINT-1-1 Node 1 in NSET SETREFPOINT1 Reaction force: RF1 PI: REFPOINT-2-1 Node 1 in NSET SETREFPOINT2	
40	Reaction force: RF1 PI: REFPOINT-3-1 Node 1 in NSET SETREFPOINT3 Reaction force: RF2 PI: REFPOINT-1-1 Node 1 in NSET SETREFPOINT1 Reaction force: RF2 PI: REFPOINT-2-1 Node 1 in NSET SETREFPOINT2 Reaction force: RF2 PI: REFPOINT-3-1 Node 1 in NSET SETREFPOINT3 Reaction force: RF3 PI: REFPOINT-3-1 Node 1 in NSET SETREFPOINT3 Reaction force: RF3 PI: REFPOINT-1-1 Node 1 in NSET SETREFPOINT1 Reaction force: RF3 PI: REFPOINT-2-1 Node 1 in NSET SETREFPOINT2 Reaction force: RF3 PI: REFPOINT-3-1 Node 1 in NSET SETREFPOINT2 Reaction force: RF3 PI: REFPOINT-3-1 Node 1 in NSET SETREFPOINT2 Reaction force: RF3 PI: REFPOINT-3-1 Node 1 in NSET SETREFPOINT2	
00	Total time in which creep was active: CRPTIME for Whole Model Save As Plot Dismiss	-
20. –		
10	The $C_{55}^{*}(t)$ was calculated as $C_{55}^{*}(t) = RF_{1}(t)/(A_{2}\bar{\varepsilon}_{12})$ where $RF_{1}(t)$ is the variation of 1 component of the reaction force of Performed point 2. $\bar{\varepsilon}_{12} = \frac{1}{2}\bar{v}_{12}$	
D	reaction force of Reference point-2. $\varepsilon_{12} = \frac{1}{2}\gamma_{12}$.	8.0 10.0

Contour plot of von ses stress of unit cell der Load case 4.

Comparison with SwiftComp



Comparison with SwiftComp



Comparison with SwiftComp

