Calculation of stress relaxation stiffness of linear viscoelastic composites using ABAQUS

Tian Tang

tang298@purdue.edu

Composites Manufacturing & Simulation Center

Purdue University

Introduction

- This tutorial demonstrates a method of calculating the effective stress relaxation stiffness of linear viscoelastic composites using ABAQUS.
- A fiber reinforced polymer matrix composite is used as an example in which the polymer is assumed to be *linear viscoelastic* behavior while the fiber is linear elastic material.
- The present methodology can be readily extended to other types of composites such as particle reinforced composites, chopped fiber reinforced composites, etc.

Periodic boundary conditions

The composites can be idealized as assembly of many periodic unit cells to which the periodic boundary conditions are consequently applied, which means that the deformation mode in each unit cell are identical and there is no gap or overlap between the adjacent unit cells. The periodic boundary conditions are represented as

$$u_i = \bar{\varepsilon}_{ij} x_j + v_i \tag{1}$$

where $\bar{\varepsilon}_{ij}$ is the average strain; v_i is the periodic part of the displacement components also called local fluctuation on the boundary surfaces. The displacements on a pair of opposite boundary surfaces are given by

$$u_i^{k+} = \bar{\varepsilon}_{ij} x_j^{k+} + v_i^{k+} \tag{2}$$

$$u_i^{k-} = \bar{\varepsilon}_{ij} x_j^{k-} + v_i^{k-} \tag{3}$$

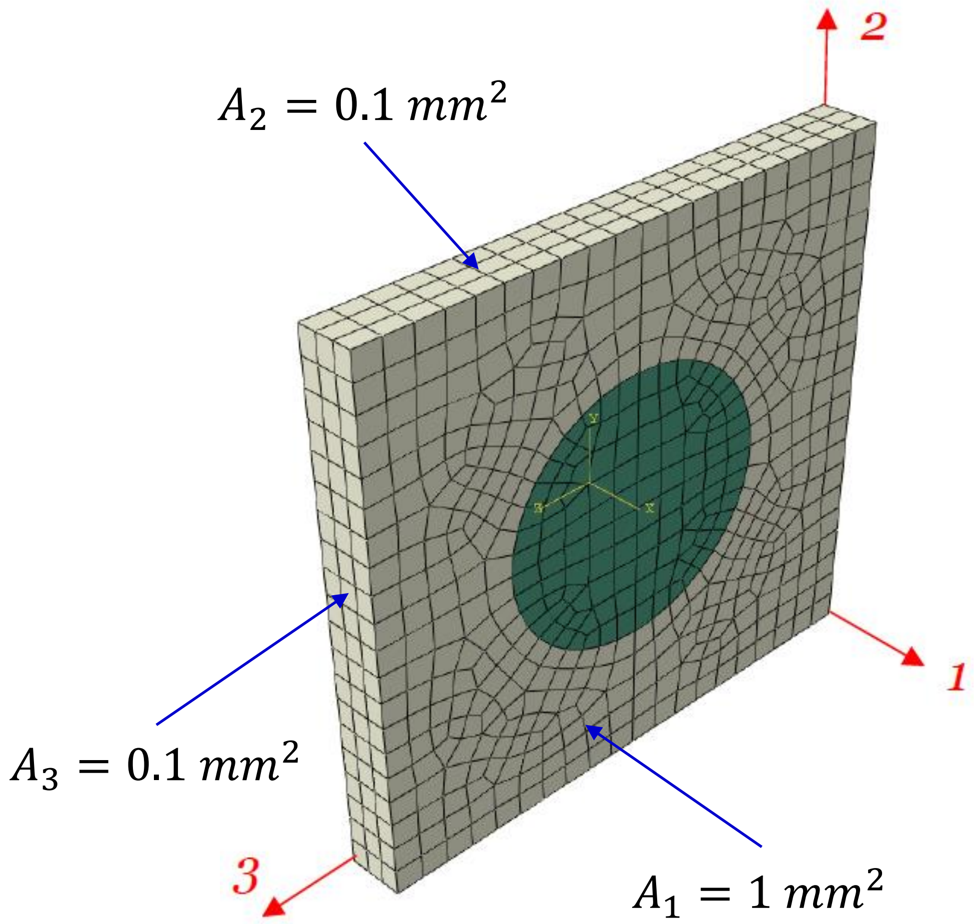
where "$k+$" denotes along the positive x_j direction while "$k-$" means along the negative x_j direction. Since the periodic parts v_i^{k+} and v_i^{k-} are identical on the two opposite boundary surfaces of a periodic unit cell, the difference of Eq. (2) and (3) is obtained as

$$u_i^{k+} - u_i^{k-} = \bar{\varepsilon}_{ij}\left(x_j^{k+} - x_j^{k-}\right) = \bar{\varepsilon}_{ij} \Delta x_j \tag{4}$$

where Δx_j is actually the edge length of the unit cell.

- The periodic boundary conditions described in Eq. (4) are implemented into a python script.
- The periodic boundary conditions described in Eq. (4) were applied to the unit cell by coupling opposite nodes on corresponding opposite boundary surfaces. In actual manipulation, three reference points are first created and their displacements are assigned as $\bar{\varepsilon}_{ij}\Delta x_j$.

Unit cell of a fiber reinforced composite



- In this study, the fiber is assumed to be of circular shape and in square array.
- The unit cell model is meshed by C3D8R elements.
- Sweep mesh technique was used in order to obtain periodic mesh on opposite boundary surfaces, which means that the meshes on opposite boundary surfaces are identical.
- In the present study, the fiber direction is along *-1*. The edge length of the unit cell along *1*, *2*, and *3* direction are respectively $\Delta x_1 = 0.1$ mm and $\Delta x_2 = \Delta x_3 = 1$ mm.

Material properties of the constituents

In the present example, the **ABAQUS** unit cell model was used to calculate the effective stress relaxation stiffness and creep compliance of the glass fiber reinforced polymer matrix composites.

- The **glass fibers** are isotropic and linear elastic materials with Young's modulus and Poisson's ratio being 80,000 MPa and 0.3 respectively. The volume fraction of the fibers is $\mathrm{vof} = 20\%$.
- The elastic relaxation modulus of the isotropic and linear viscoelastic **polymer** materials can be expressed using Prony series in the following way:

$$E(t) = E_0\left(1 - \sum_{k=1}^{n} g_k\left(1 - e^{-t/\tau_k}\right)\right) = E_\infty + \sum_{k=1}^{n} E_i e^{-t/\tau_k}$$

where E_0 is the instantaneous Young's modulus and also given by

$E_0 = E_\infty + \sum_{k=1}^{n} E_i = E_\infty + \sum_{k=1}^{n} E_0 g_k$ with E_∞ being the long-term Young's modulus;

τ_k is the time relaxation material parameter.

Material properties of the constituents (cont.)

Table 1. Relaxation times and Prony coefficients for PMT-F4 epoxy.*

i	E_i, MPa	ρ_i, s
∞	1000	— —
1	224.1	1.0e + 3
2	450.8	1.0e + 5
3	406.1	1.0e + 6
4	392.7	1.0e + 7
5	810.4	1.0e + 8
6	203.7	1.0e + 9
7	1486.0	1.0e + 10

* Kawai Kwok and Sergio Pellegrino, "Micromechanics models for viscoelastic plain-weave composite tape springs", AIAA Journal, Vol. 55, No. 1, January 2017.

In the present example, the material properties of the polymer matrix is shown in left table. The Poisson's ratio of the polymer is 0.33.

$$E_0 = E_\infty + \sum_{i=1}^{7} E_i = 4973.8 \text{ Mpa}$$

$$g_1 = \frac{E_1}{E_0} = 0.045056094; \ g_2 = \frac{E_2}{E_0} = 0.090634927$$

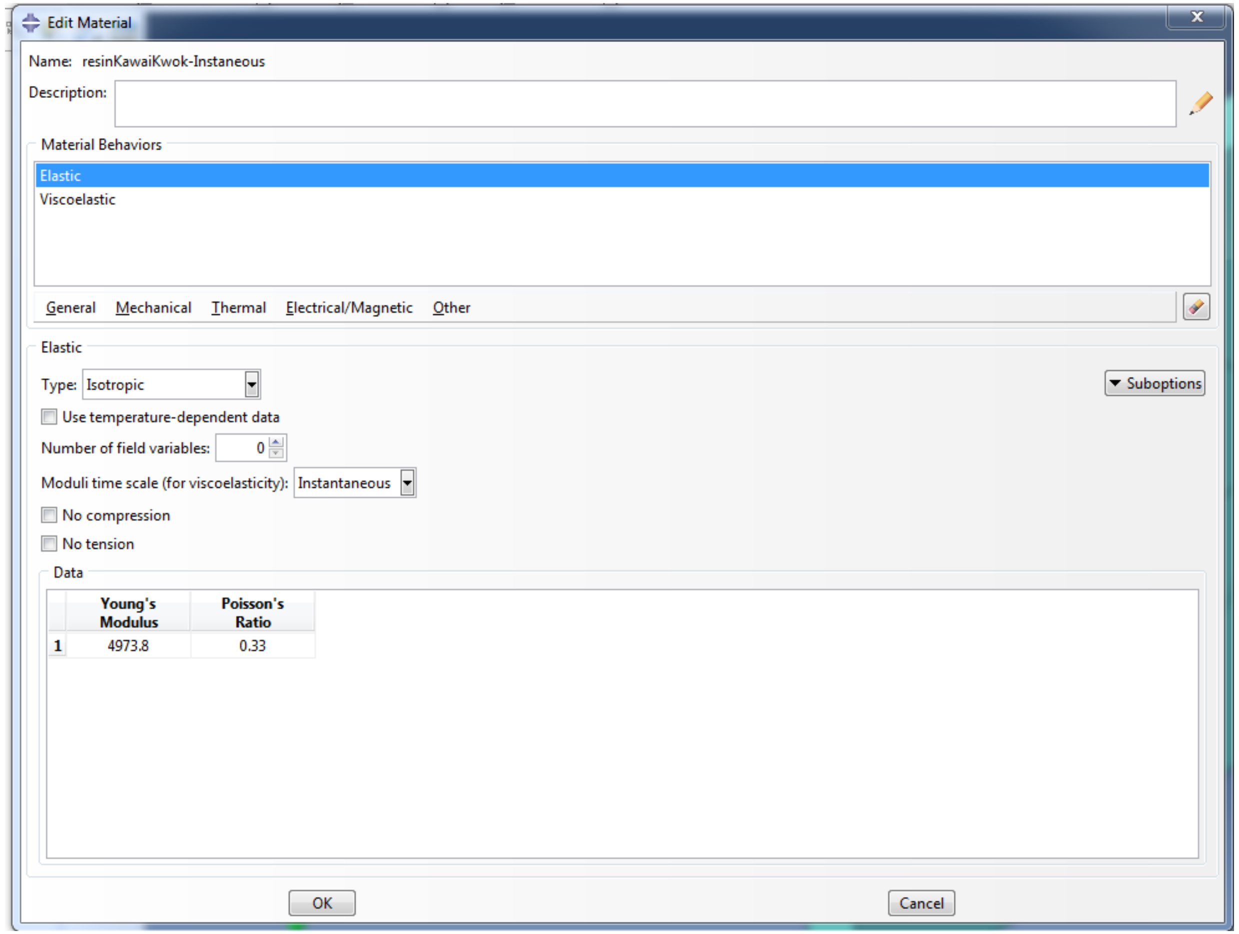
$$g_3 = \frac{E_3}{E_0} = 0.081647835; \ g_4 = \frac{E_4}{E_0} = 0.0789553717$$

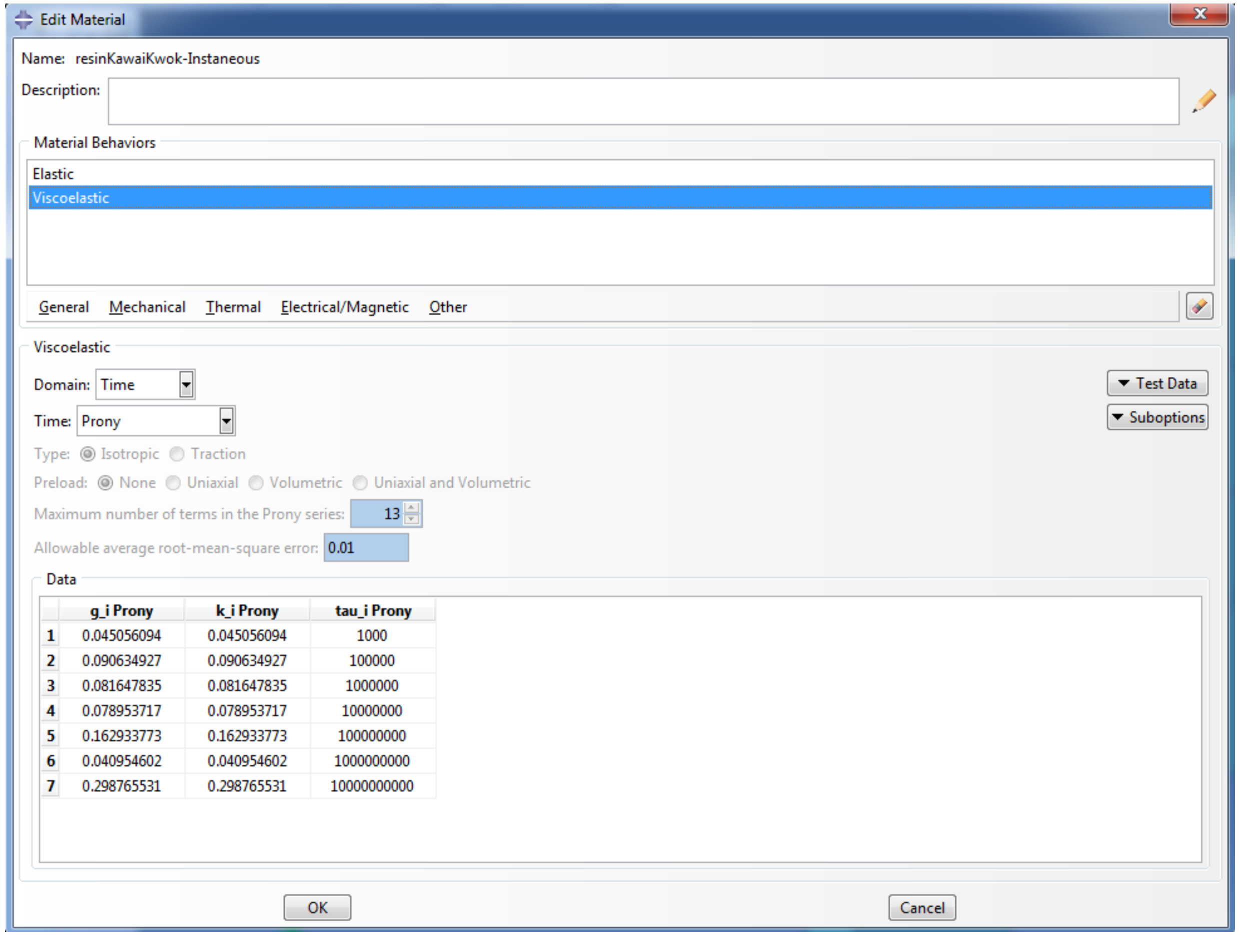
$$g_5 = \frac{E_5}{E_0} = 0.162933773; \ g_6 = \frac{E_6}{E_0} = 0.040954602$$

$$g_7 = \frac{E_7}{E_0} = 0.045056094$$

where g_i is the dimensionless Young's modulus.

Material inputs of the polymer in ABAQUS unit cell model





Note that $g_i = k_i$ **with** k_i **being the dimensionless bulk modulus.**

Effective stress relaxation stiffness

- The effective properties of fiber reinforced composites with the fibers in square array possess square symmetry. The effective stress relaxation stiffness matrix can be expressed as:

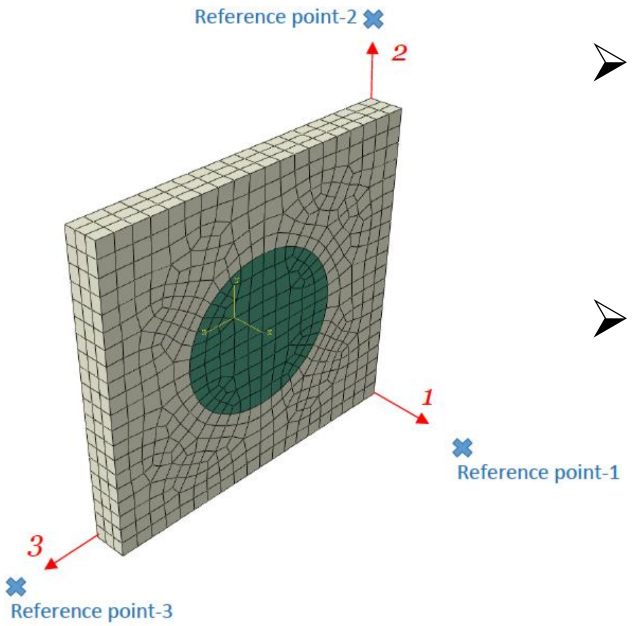
$$\begin{Bmatrix} \bar{\sigma}_{11}(t) \\ \bar{\sigma}_{22}(t) \\ \bar{\sigma}_{33}(t) \\ \bar{\sigma}_{23}(t) \\ \bar{\sigma}_{12}(t) \\ \sigma_{13}(t) \end{Bmatrix} = \begin{bmatrix} C^*_{11}(t) & C^*_{12}(t) & C^*_{12}(t) & 0 & 0 & 0 \\ C^*_{12}(t) & C^*_{22}(t) & C^*_{23} & 0 & 0 & 0 \\ C^*_{12}(t) & C^*_{23}(t) & C^*_{22} & 0 & 0 & 0 \\ 0 & 0 & 0 & C^*_{44}(t) & 0 & 0 \\ 0 & 0 & 0 & 0 & C^*_{55}(t) & 0 \\ 0 & 0 & 0 & 0 & 0 & C^*_{55}(t) \end{bmatrix} \begin{Bmatrix} \bar{\varepsilon}^{\mathrm{cst}}_{11} \\ \bar{\varepsilon}^{\mathrm{cst}}_{22} \\ \bar{\varepsilon}^{\mathrm{cst}}_{33} \\ \bar{\gamma}^{\mathrm{cst}}_{23} \\ \bar{\gamma}^{\mathrm{cst}}_{12} \\ \bar{\gamma}^{\mathrm{cst}}_{13} \end{Bmatrix}$$

Where “cst” means constant values that do not vary with time but may change with position.

Load cases

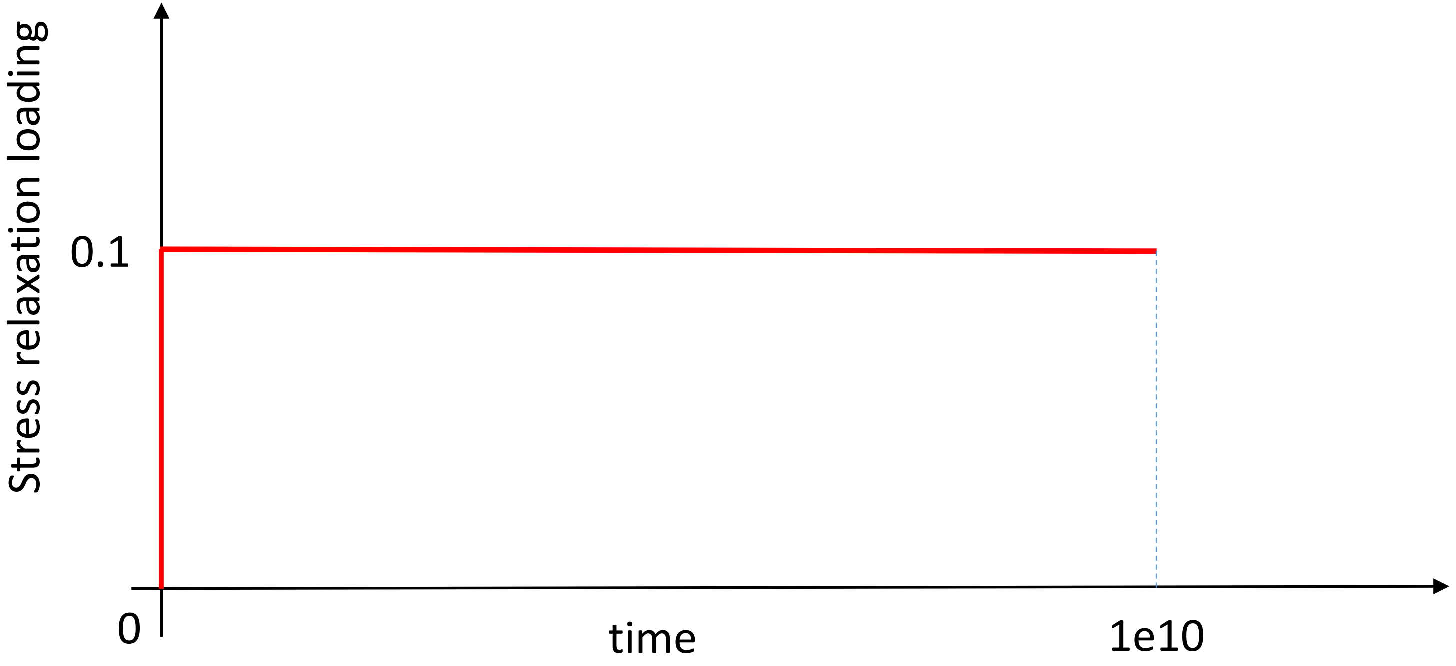
In order to calculate the full set of stress relaxation stiffness of the linear viscoelastic composites, four load cases are applied:

- **Load case 1**: The constant macroscopic strain $\bar{\varepsilon}_{11} = 0.1$ along *1* direction was applied by prescribing the *1* direction displacement of Reference point-1 as $u_1 = 0.01$. All other mechanical strains are set to zero.
- **Load case 2**: The constant macroscopic strain $\bar{\varepsilon}_{22} = 0.1$ along *2* direction was applied by prescribing the *2* direction displacement of Reference point-2 as $u_2 = 0.1$. All other mechanical strains are set to zero.



- **Load case 3**: The constant macroscopic transverse shear strain $\bar{\gamma}_{23} = 0.1$ was applied by prescribing the *3* direction displacement of Reference point-2 as $u_3 = 0.1$ and *2* direction displacement of Reference point-3 as $u_2 = 0.1$. All other mechanical strains are set to zero.
- **Load case 4**: The constant macroscopic longitudinal shear strain $\bar{\gamma}_{12} = 0.1$ was applied by prescribing the *2* direction displacement of Reference point-1 as $u_2 = 0.01$ and *1* direction displacement of Reference point-2 as $u_1 = 0.1$. All other mechanical strains are set to zero.

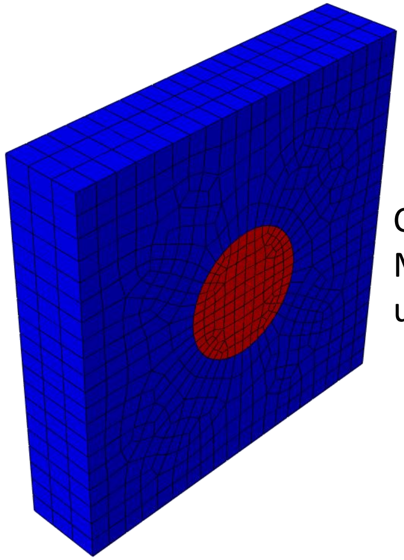
Stress relaxation loading



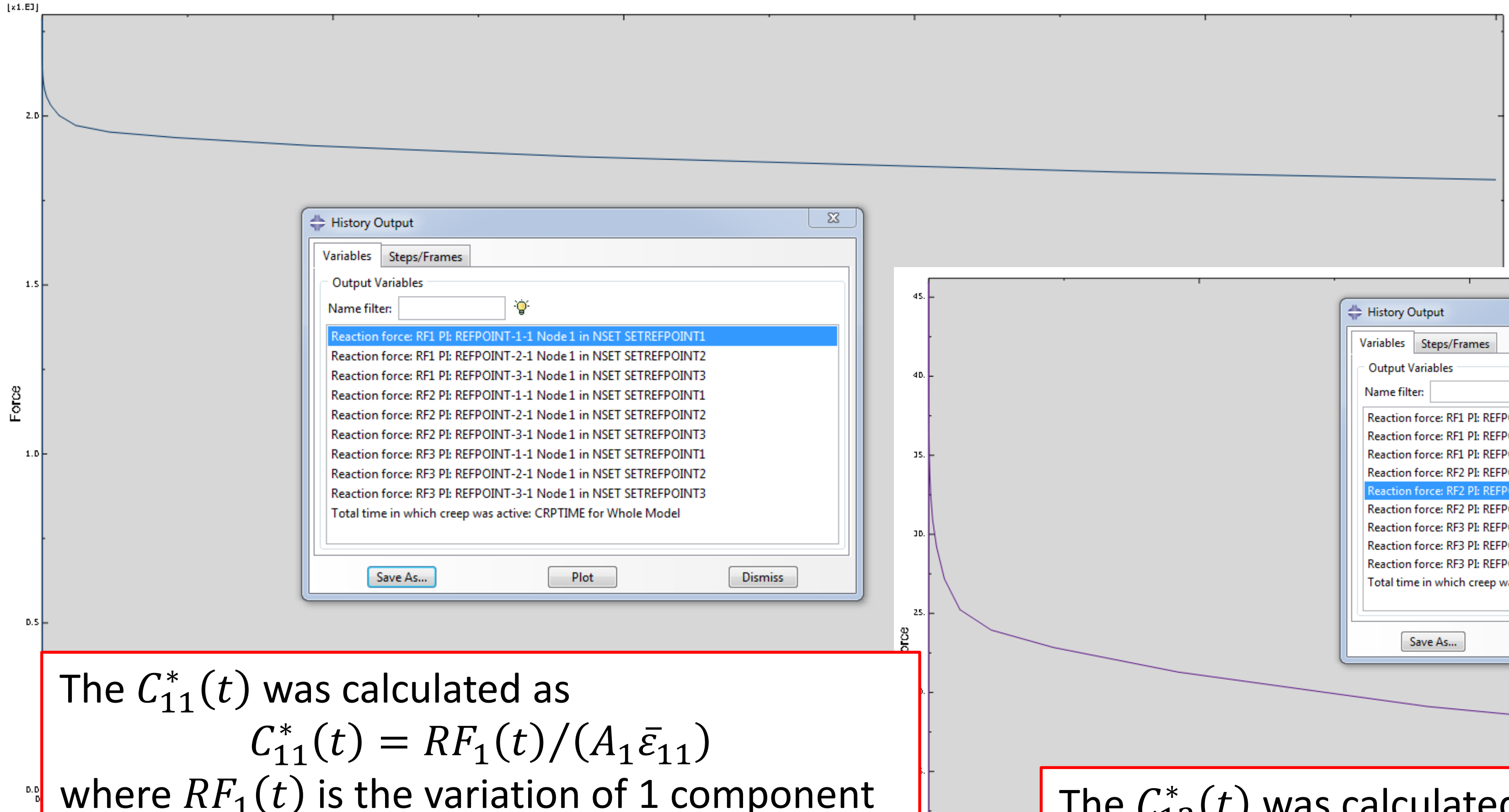
The constant strain was applied since $t = 0$.

Calculation of $C_{11}^*(t)$ and $C_{12}^*(t)$

The calculation of $C_{11}^*(t)$ and $C_{12}^*(t)$ are calculated under **Load case 1**.



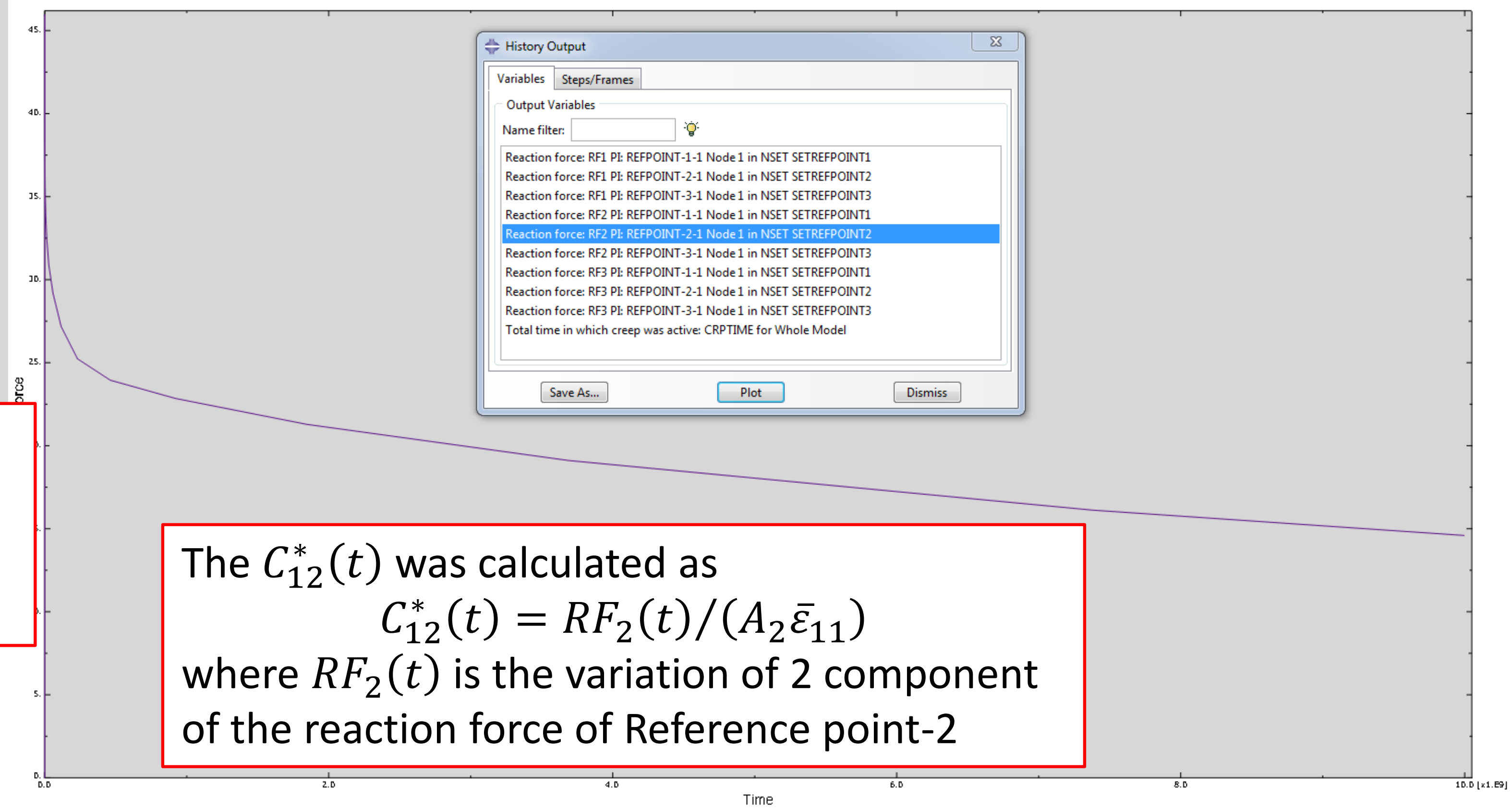
Contour plot of von Mises stress of unit cell under Load case 1.



The $C_{11}^*(t)$ was calculated as

$$C_{11}^*(t) = RF_1(t)/(A_1\bar{\varepsilon}_{11})$$

where $RF_1(t)$ is the variation of 1 component of the reaction force of Reference point-1



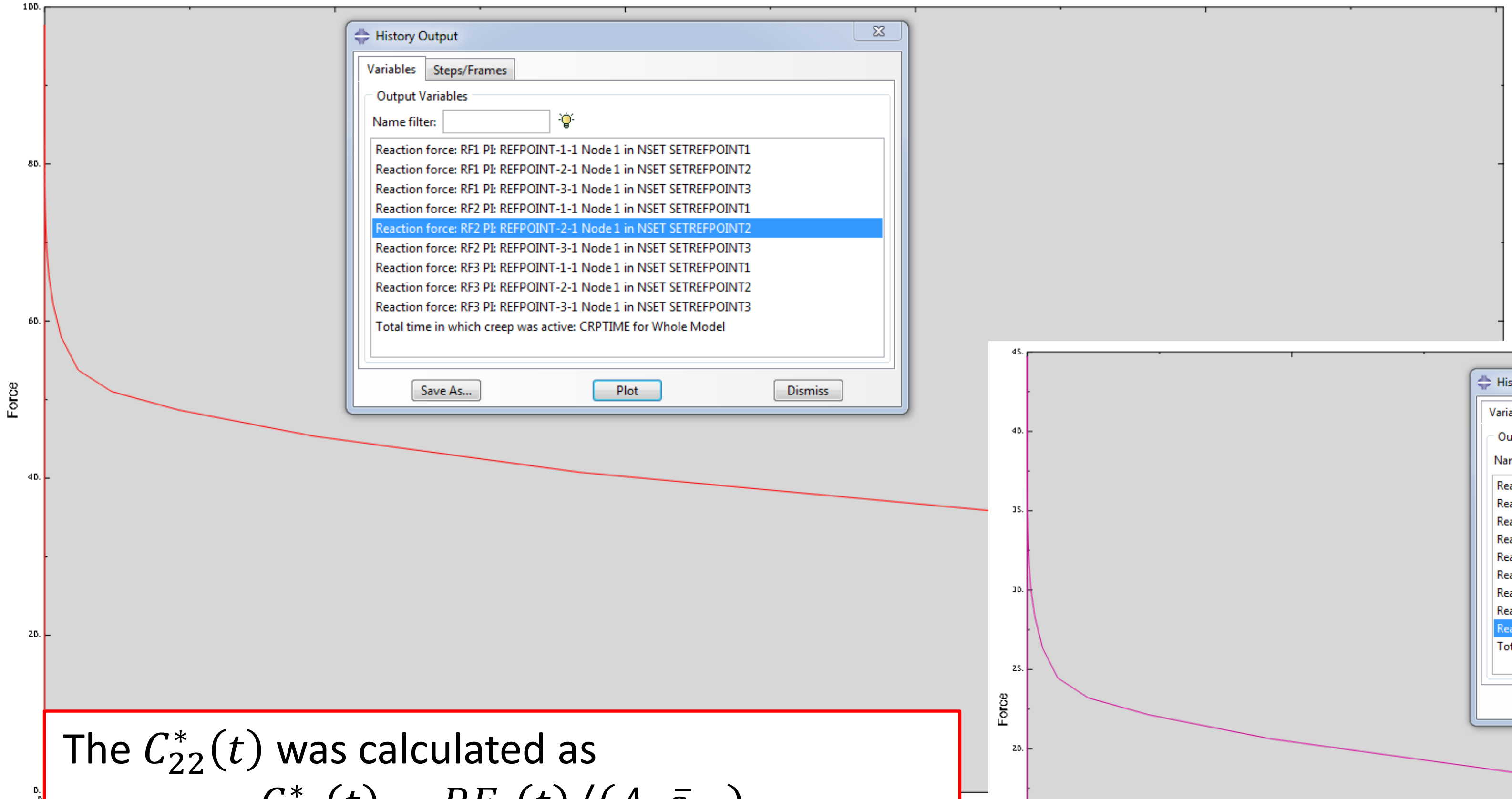
The $C_{12}^*(t)$ was calculated as

$$C_{12}^*(t) = RF_2(t)/(A_2\bar{\varepsilon}_{11})$$

where $RF_2(t)$ is the variation of 2 component of the reaction force of Reference point-2

Calculation of $C^*_{22}(t)$ and $C^*_{23}(t)$

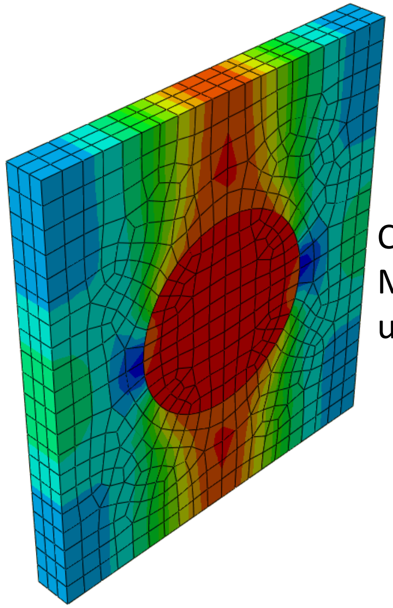
The calculation of $C^*_{22}(t)$ and $C^*_{23}(t)$ are calculated under **Load case 2**.



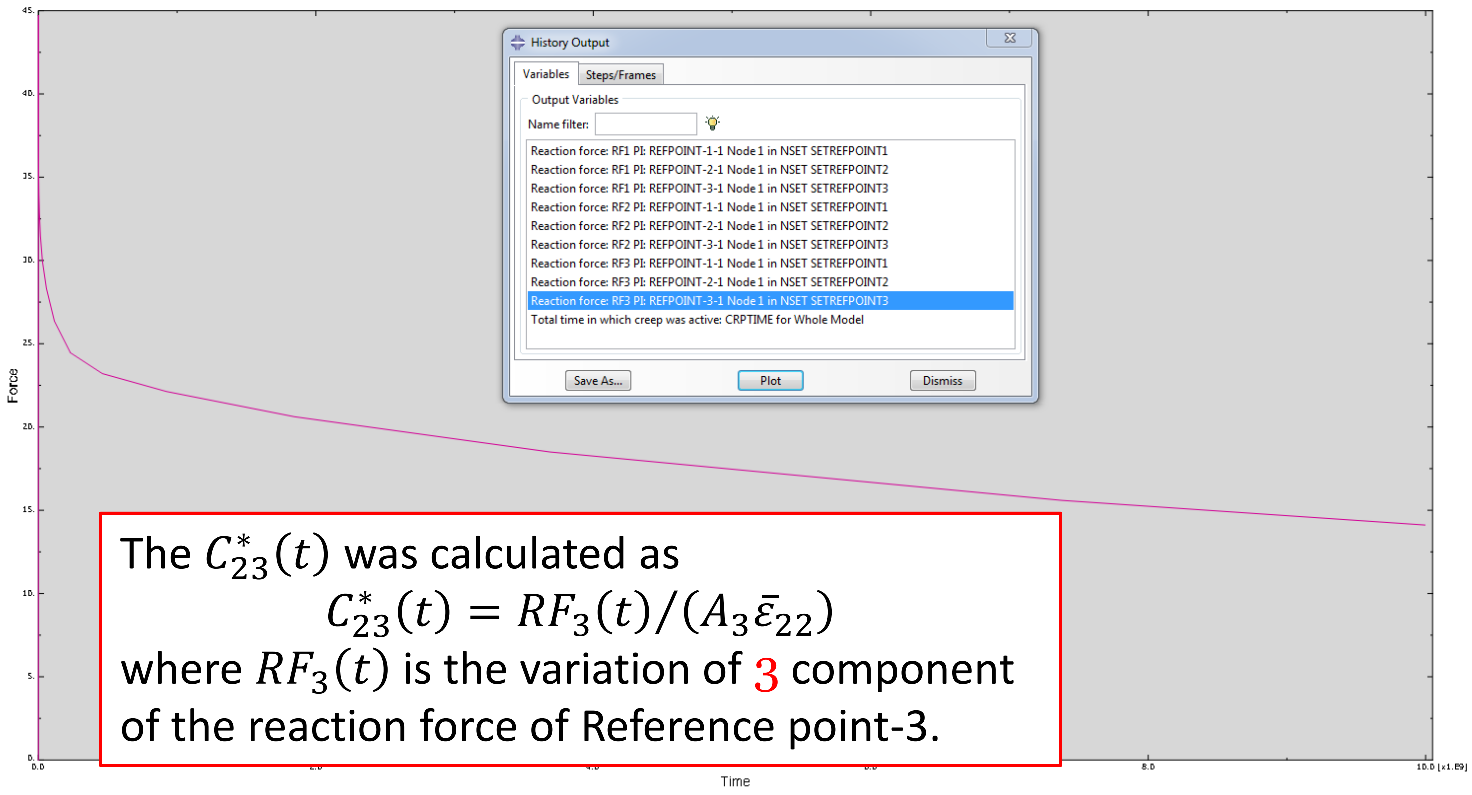
The $C^*_{22}(t)$ was calculated as

$$C^*_{22}(t) = RF_2(t)/(A_2\bar{\varepsilon}_{22})$$

where $RF_2(t)$ is the variation of 2 component of the reaction force of Reference point-2.



Contour plot of von Mises stress of unit cell under Load case 2.



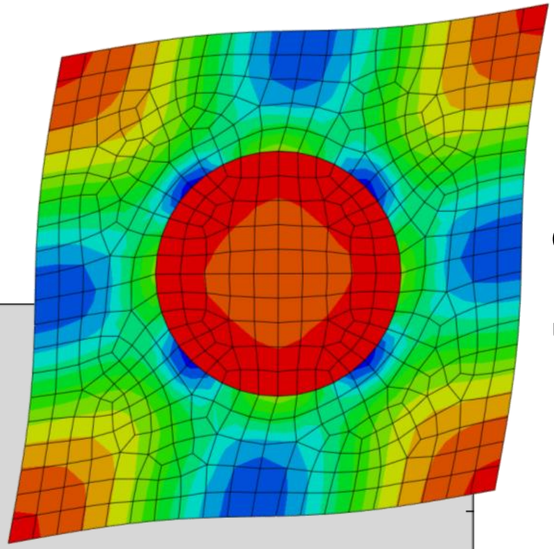
The $C^*_{23}(t)$ was calculated as

$$C^*_{23}(t) = RF_3(t)/(A_3\bar{\varepsilon}_{22})$$

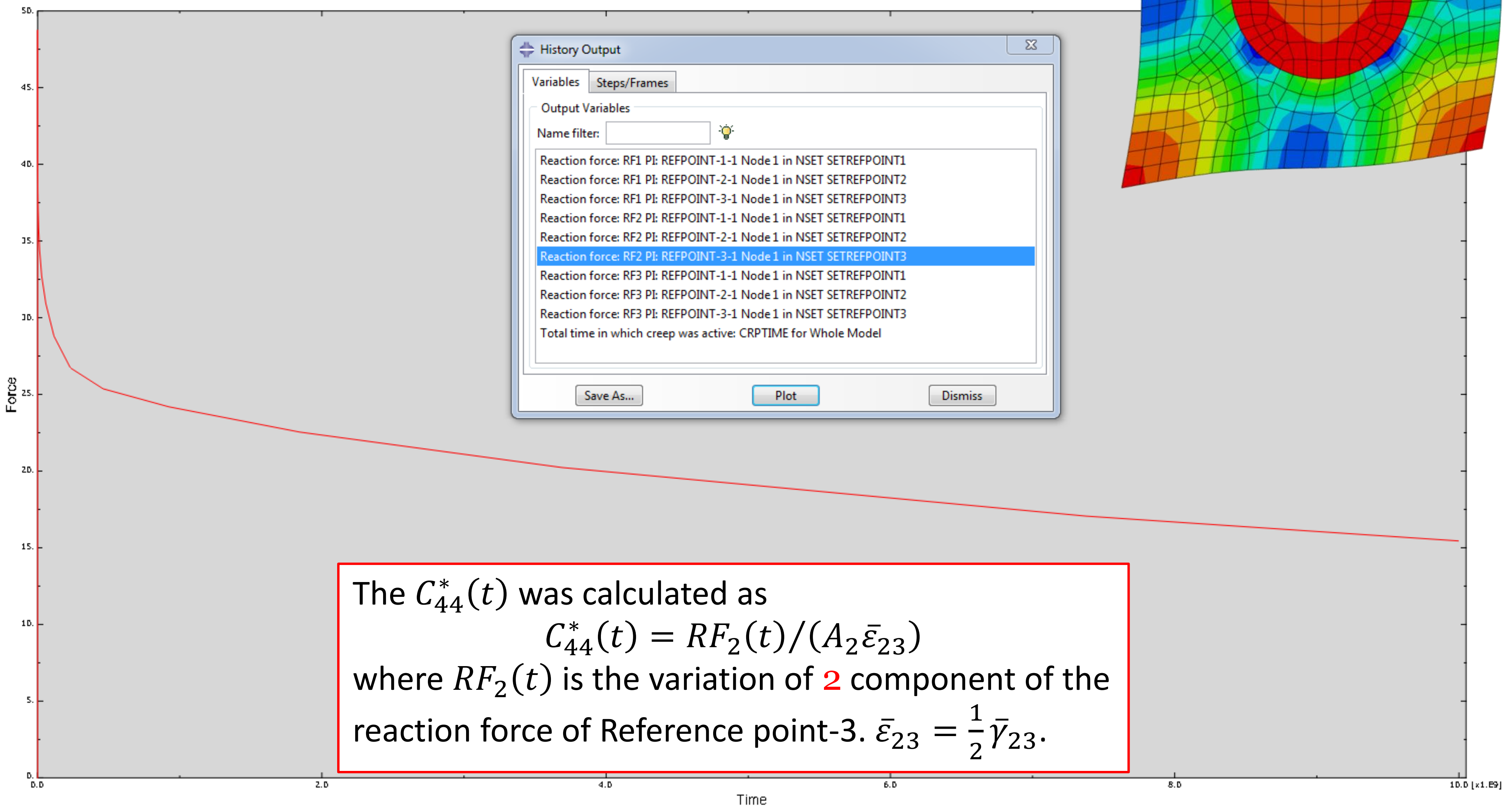
where $RF_3(t)$ is the variation of 3 component of the reaction force of Reference point-3.

Calculation of $C^*_{44}(t)$

The calculation of $C^*_{44}(t)$ is calculated under **Load case 3**.



Contour plot of von Mises stress of unit cell under Load case 3.



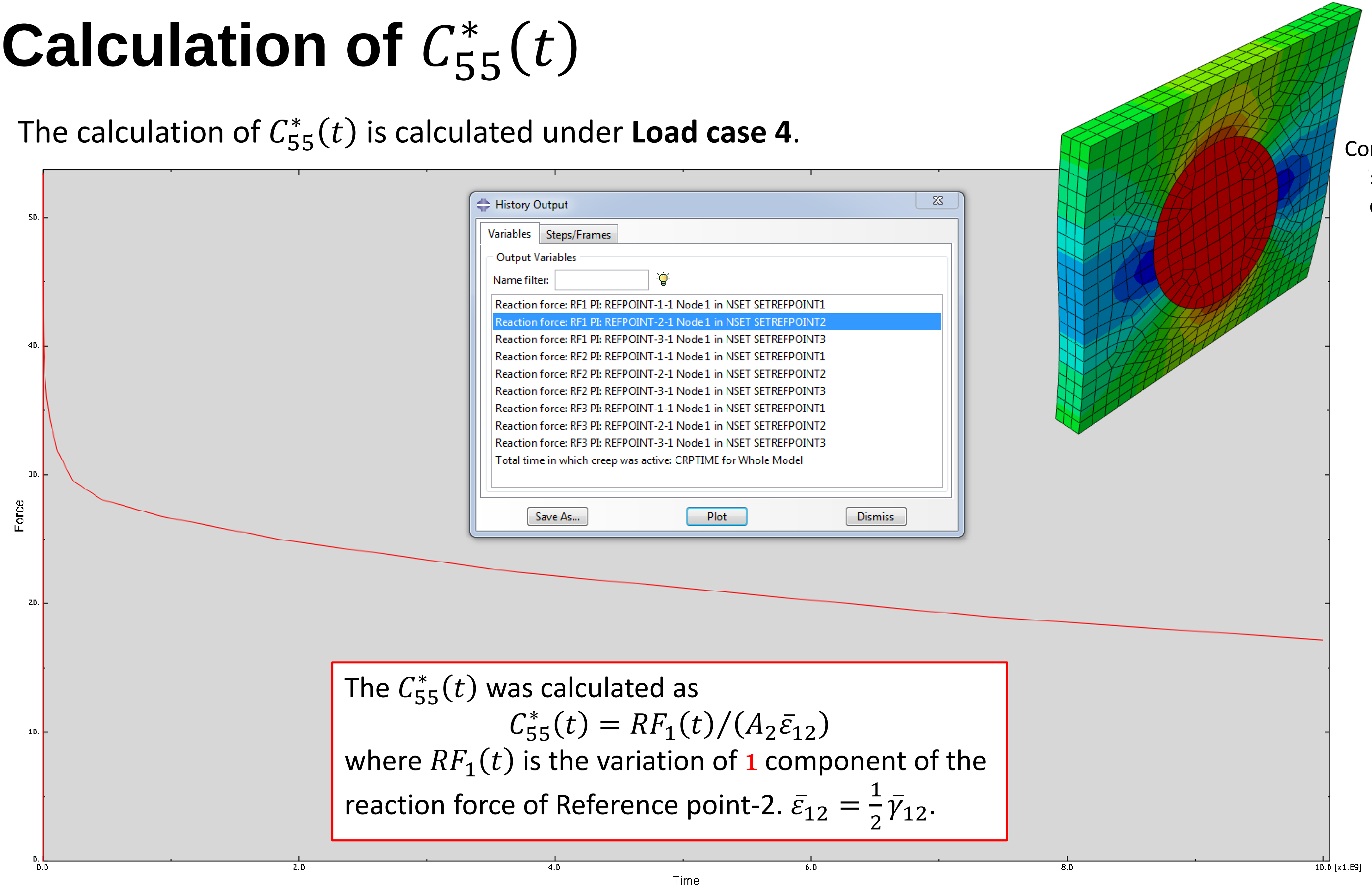
The $C^*_{44}(t)$ was calculated as

$$C^*_{44}(t) = RF_2(t)/(A_2\bar{\varepsilon}_{23})$$

where $RF_2(t)$ is the variation of 2 component of the reaction force of Reference point-3. $\bar{\varepsilon}_{23} = \frac{1}{2}\bar{\gamma}_{23}$.

Calculation of $C^*_{55}(t)$

The calculation of $C^*_{55}(t)$ is calculated under **Load case 4**.



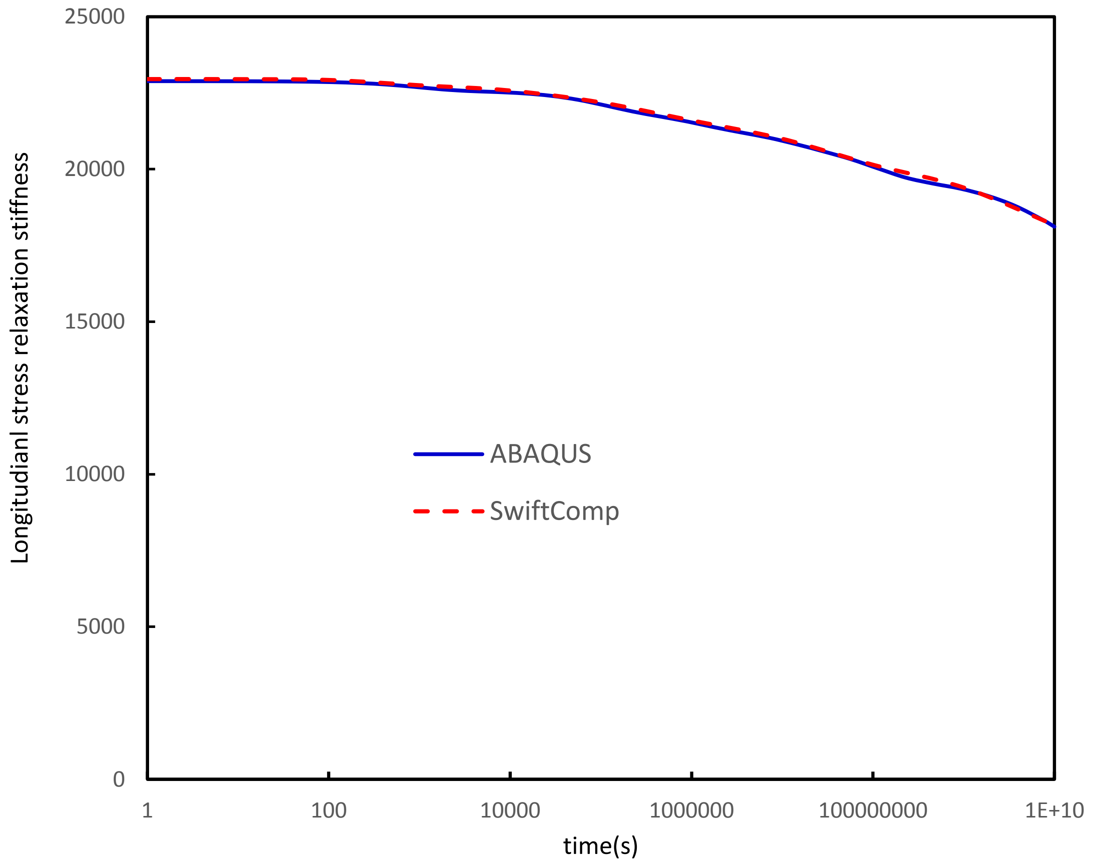
Contour plot of von
ses stress of unit cell
der Load case 4.

The $C^*_{55}(t)$ was calculated as

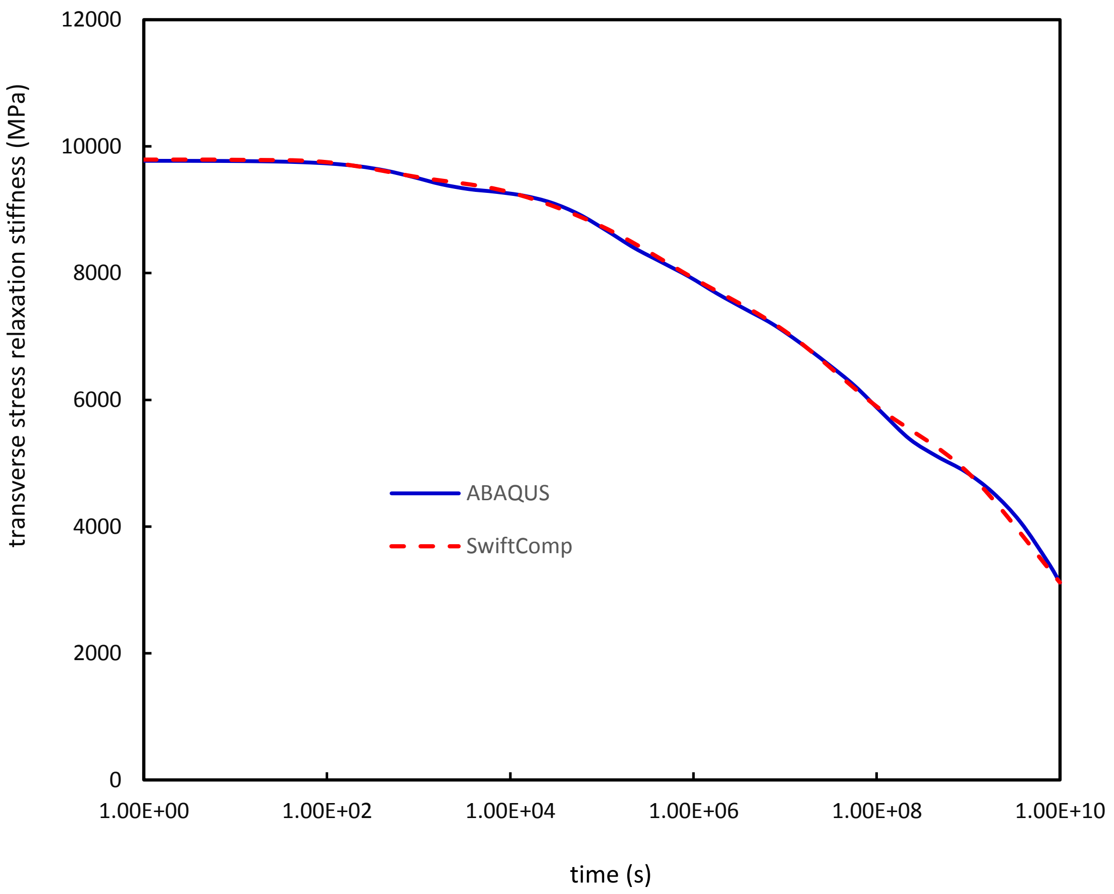
$$C^*_{55}(t) = RF_1(t)/(A_2\bar{\varepsilon}_{12})$$

where $RF_1(t)$ is the variation of 1 component of the reaction force of Reference point-2. $\bar{\varepsilon}_{12} = \frac{1}{2}\bar{\gamma}_{12}$.

Comparison with SwiftComp

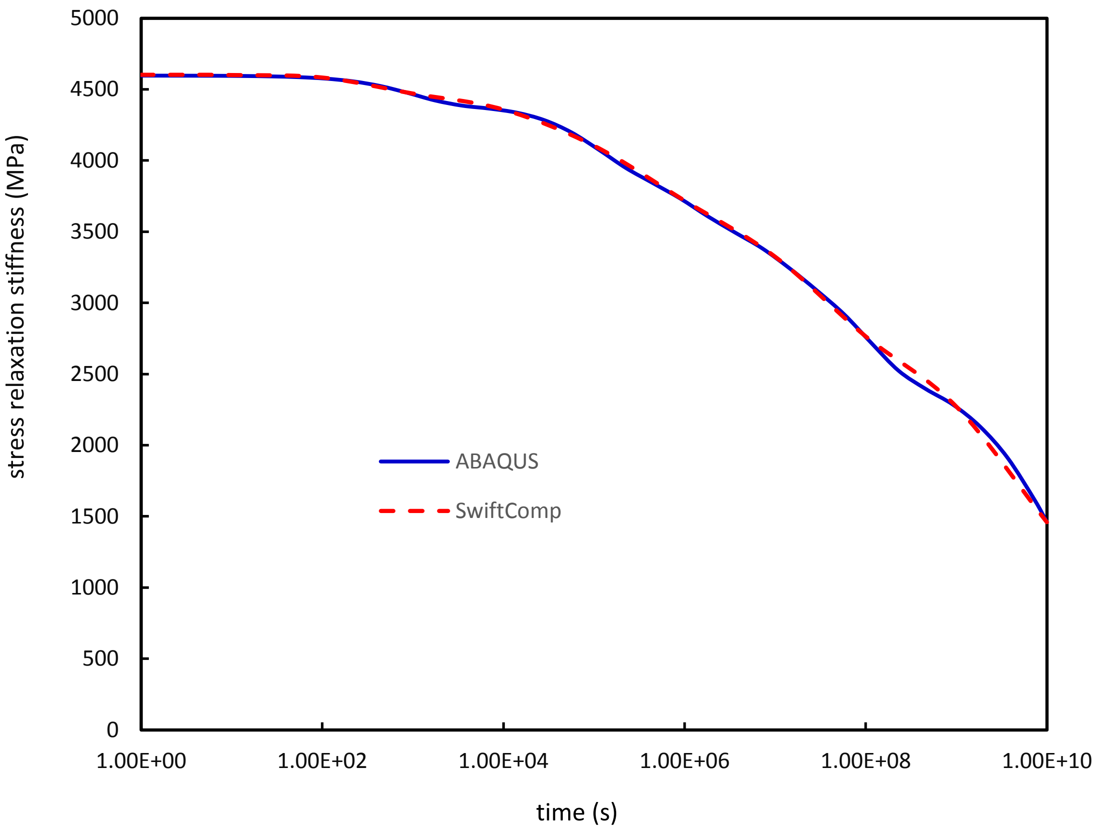


$C^*_{11}(t)$

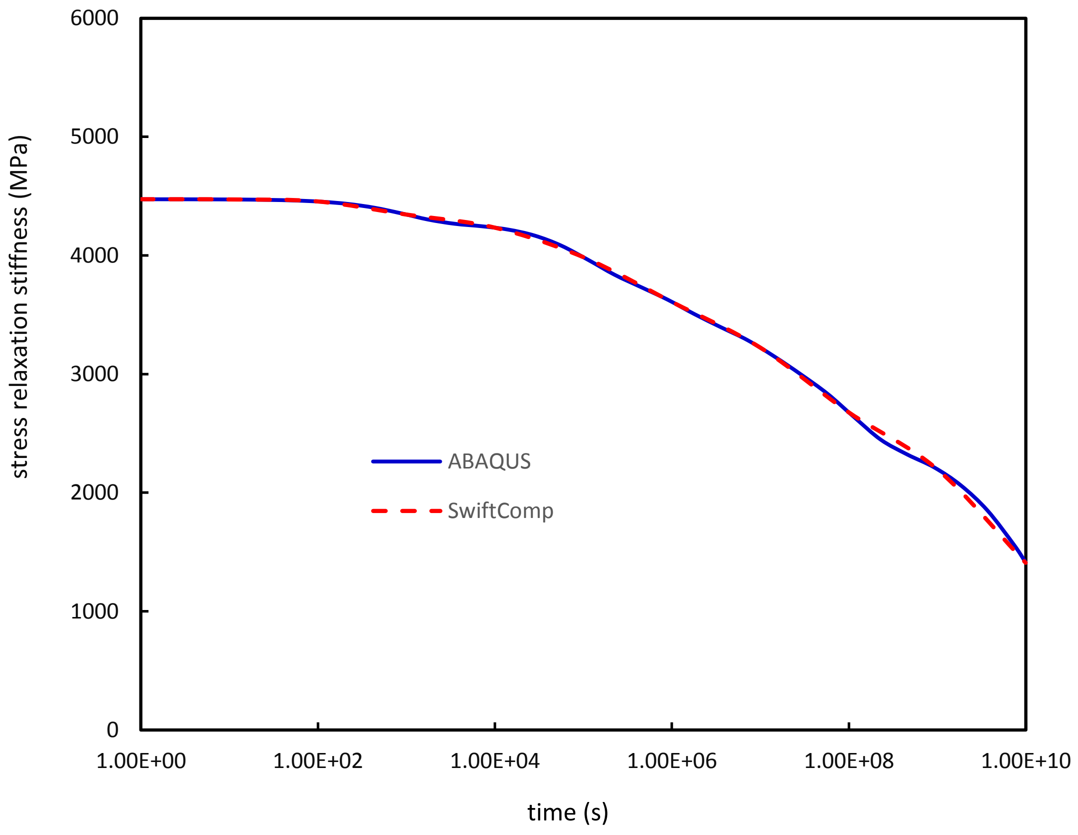


$C^*_{22}(t)$

Comparison with SwiftComp

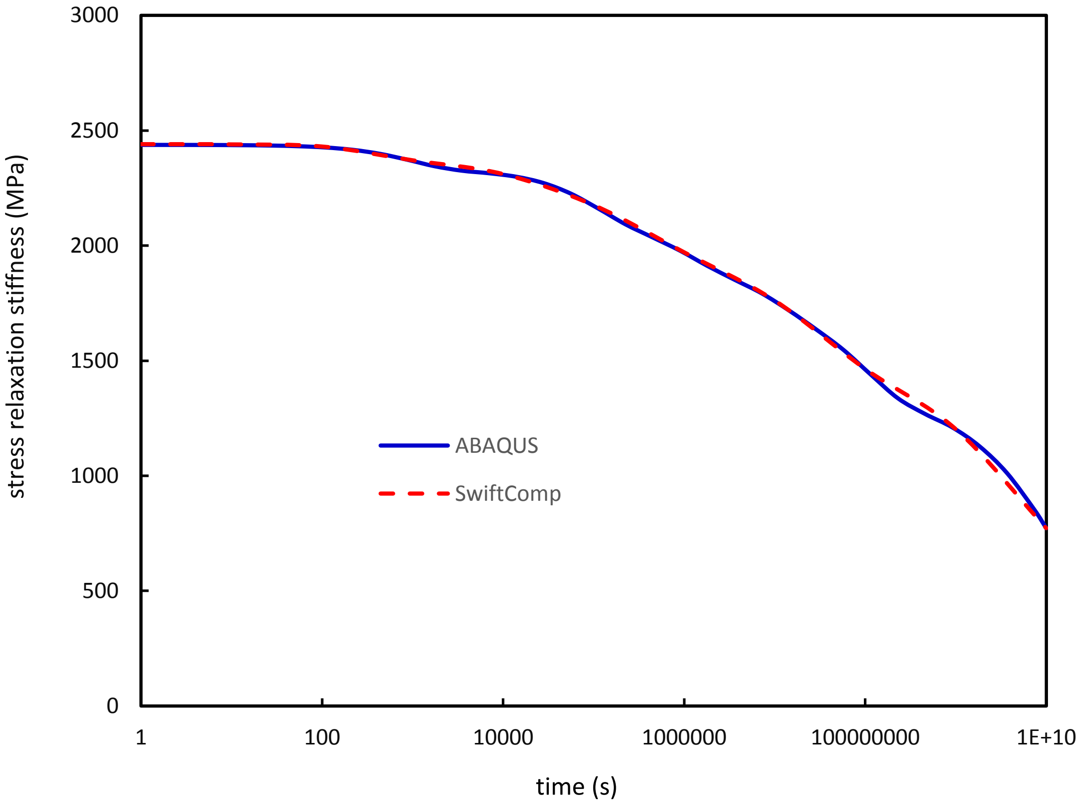


$C_{12}^*(t)$

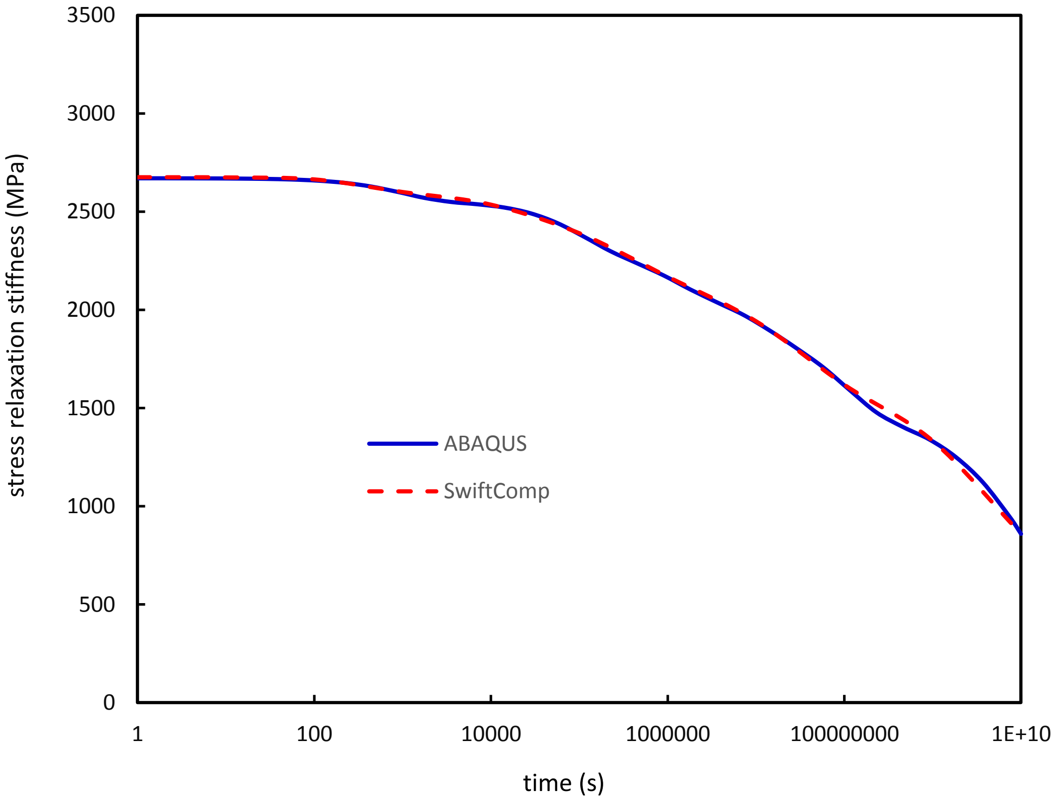


$C_{23}^*(t)$

Comparison with SwiftComp



$C_{44}^{*}(t)$



$C_{55}^{*}(t)$