MSG-BASED MULTISCALE MODELING FOR BEAMS

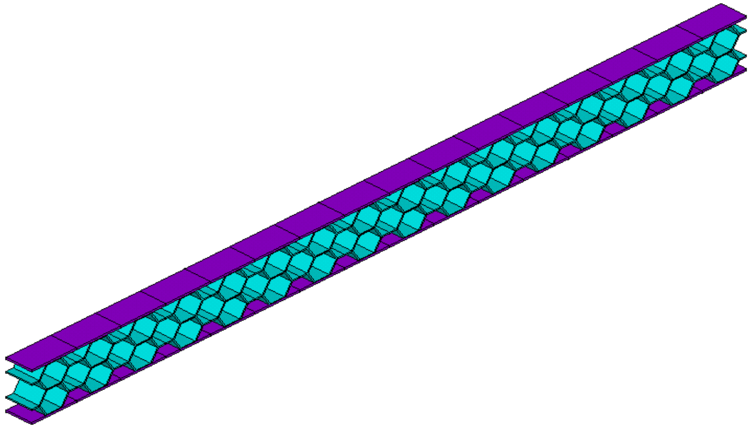
Wenbin Yu

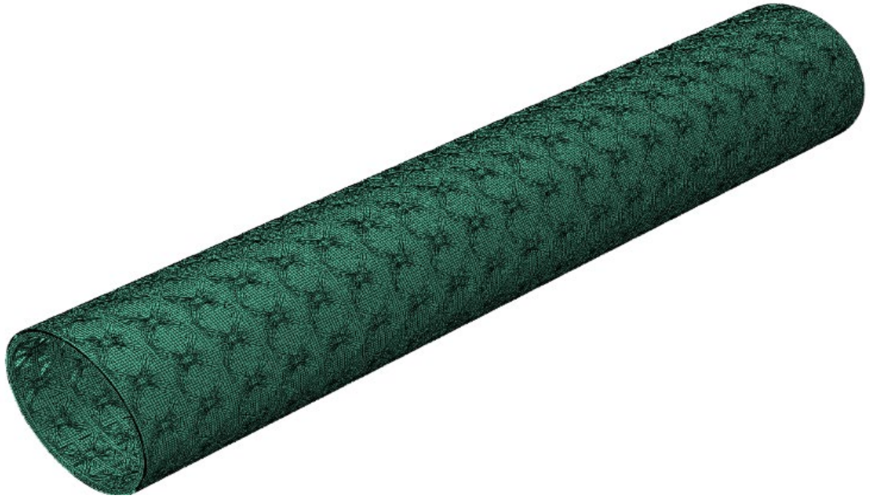
Professor, Purdue/AAE, Purdue/CMSC
Director, cdmHUB
Associate Director, IACMI/cvfHUB
CTO, AnalySwift LLC

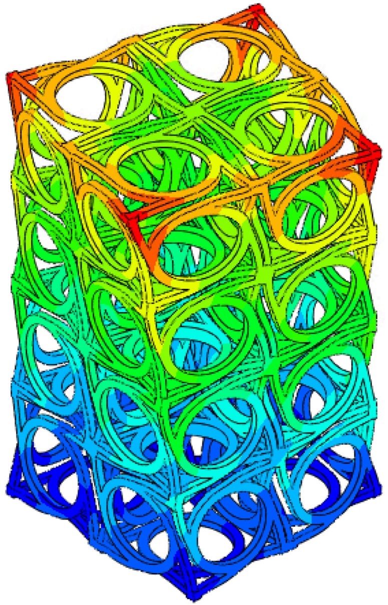
Need of Multiscale Modeling for Beams

2016, 88.4 m

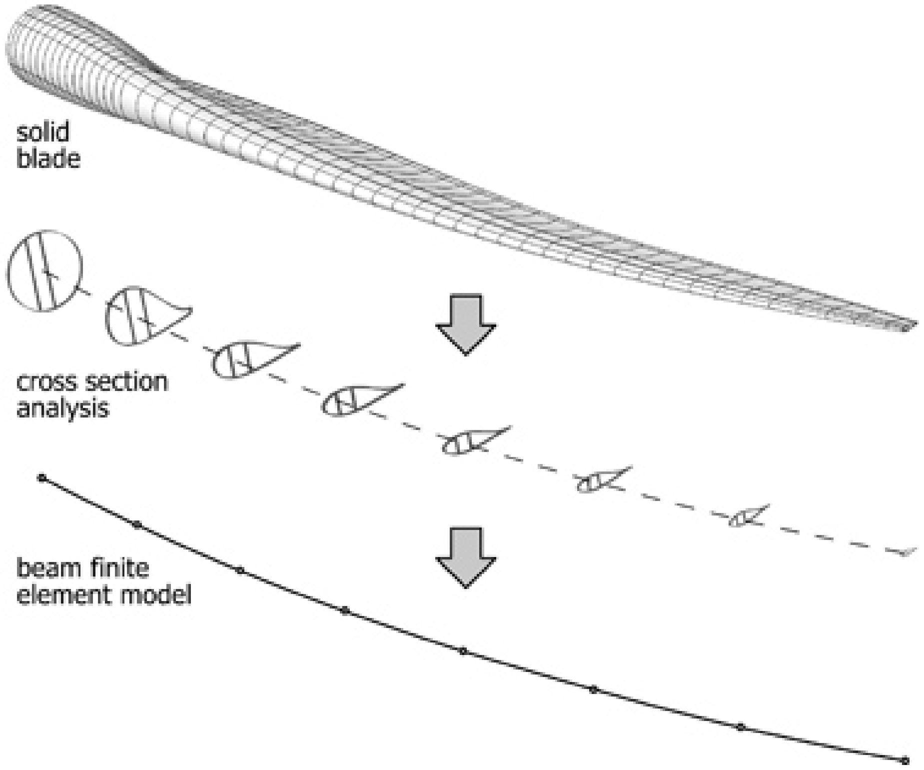

2019, 107 m







Design of Wind Turbine Blades



Courtesy of DTU Wind Energy

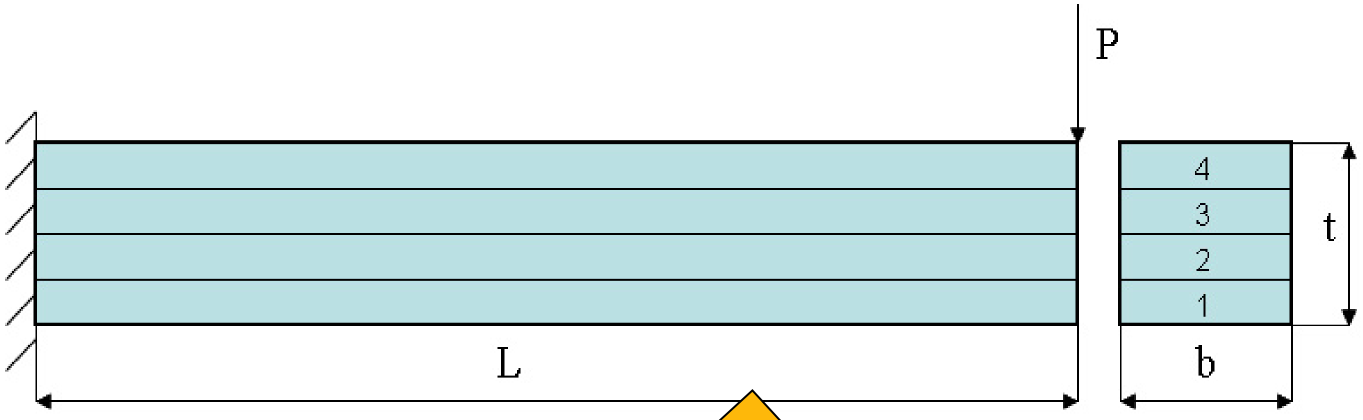
Design of a wind turbine blade

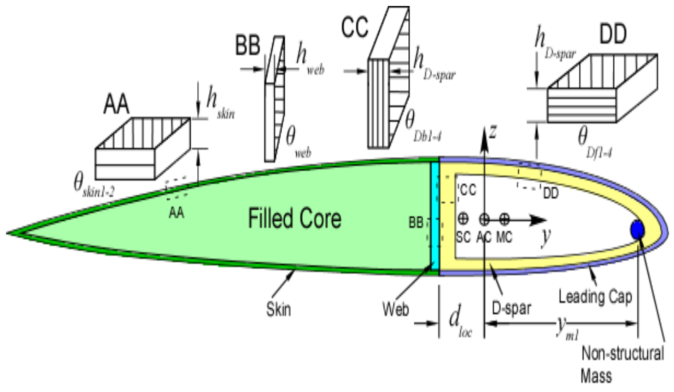
Phase 1: pre-design based on1D beam analysis together and 2D cross section analysis.

Phase 2: design with full 3D analysis of the blade.

Full 3D analysis is several orders of magnitude higher in terms of computational costs.

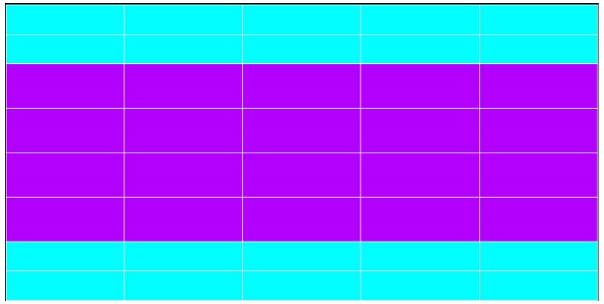
Three Essences of a Beam Theory





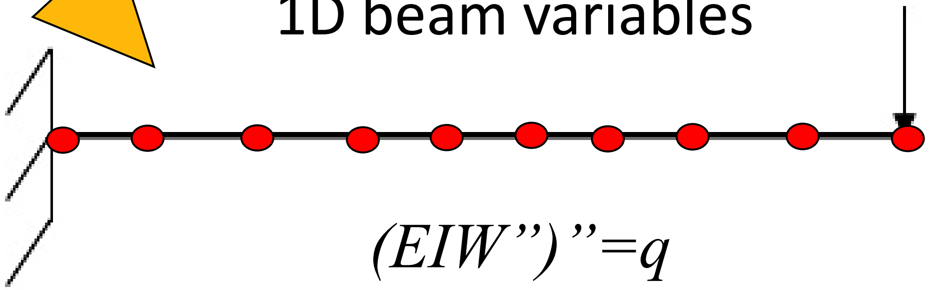
$$\sigma_x = My / I$$

Expressions of 3D fields in terms of 1D beam variables



EA, EIx, EIy, GJ,
kx GA, ky GA

Expressions to evaluate sectional properties



$$(EIW'')''=q$$

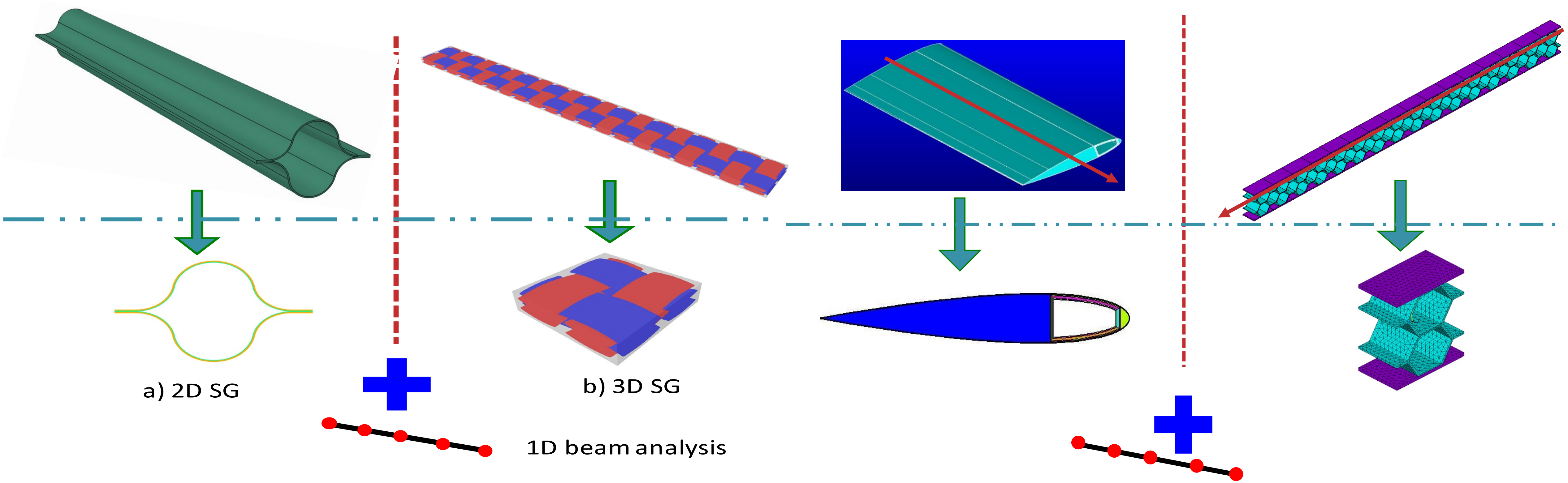
A closed set of 1D differential equations

Most beam theories are derived by assuming the cross section to deform in a specific fashion: Euler-Bernoulli, Timoshenko, Vlasov.

Traditional Beam Model

- Invoke adhoc kinematic assumptions to express the kinematics.
- Invoke unixal stress assumption to relate 3D stresses and strains.
- Define beam stress resultants in terms of 3D stresses.
- Derive equilibrium equation using the Newtonian approach or the variational approach.
- Solve the beam equations to obtain the global beam behavior including displacements, rotations, forces and moments.
- Recover 3D stresses/strains based on the global beam behavior.

MSG-based Multiscale Beam Modeling



MSG-based Multiscale Beam Modeling

- Kinematics

$$u_1(x_1,y_1,y_2,y_3) = \bar{u}_1(x_1) - \varepsilon y_3 \bar{u}_3'(x_1) - \varepsilon y_2 \bar{u}_2'(x_1) + \varepsilon w_1(x_1,y_1,y_2,y_3)$$

$$u_2(x_1,y_1,y_2,y_3) = \bar{u}_2(x_1) - \varepsilon y_3 \Phi_1(x_1) + \varepsilon w_2(x_1,y_1,y_2,y_3)$$

$$u_3(x_1,y_1,y_2,y_3) = \bar{u}_3(x_1) + \varepsilon y_2 \Phi_1(x_1) + \varepsilon w_3(x_1,y_1,y_2,y_3)$$

$$\Phi_1 = \frac{1}{2}\langle u_{3,2} - u_{2,3}\rangle \qquad \langle w_i\rangle = 0 \quad \langle w_{3|2} - w_{2|3}\rangle = 0$$

$$\bar{u}_2 = \langle u_2\rangle + \varepsilon\langle y_3\rangle\Phi_1$$

$$\bar{u}_3 = \langle u_3\rangle - \varepsilon\langle y_2\rangle\Phi_1$$

$$\bar{u}_1 = \langle u_1\rangle + \varepsilon\langle y_3\rangle\bar{u}_3' + \varepsilon\langle y_2\rangle\bar{u}_2'$$

$$\varepsilon_{11} = \epsilon_1(x_1) + \varepsilon y_3\kappa_2(x_1) - \varepsilon y_2\kappa_3(x_1) + \underbrace{w_{1|1}} + \underline{\varepsilon w_{1,1}}$$

$$\varepsilon_{22} = w_{2|2} \quad \varepsilon_{33} = w_{3|3} \quad 2\varepsilon_{23} = w_{2|3} + w_{3|2}$$

$$2\varepsilon_{12} = -\varepsilon y_3\kappa_1 + w_{1|2} + \underbrace{w_{2|1}} + \underline{\varepsilon w_{2,1}}$$

$$2\varepsilon_{13} = \varepsilon y_2\kappa_1 + w_{1|3} + \underbrace{w_{3|1}} + \underline{\varepsilon w_{3,1}}$$

MSG-based Multiscale Beam Modeling

- Kinematics

$$u_1(x_1,y_1,y_2,y_3) = \bar{u}_1(x_1) - \varepsilon y_3 \bar{u}_3'(x_1) - \varepsilon y_2 \bar{u}_2'(x_1) + \varepsilon w_1(x_1,y_1,y_2,y_3)$$

$$u_2(x_1,y_1,y_2,y_3) = \bar{u}_2(x_1) - \varepsilon y_3 \Phi_1(x_1) + \varepsilon w_2(x_1,y_1,y_2,y_3)$$

$$u_3(x_1,y_1,y_2,y_3) = \bar{u}_3(x_1) + \varepsilon y_2 \Phi_1(x_1) + \varepsilon w_3(x_1,y_1,y_2,y_3)$$

$$\Phi_1 = \frac{1}{2}\langle u_{3,2} - u_{2,3}\rangle \qquad \langle w_i\rangle = 0 \quad \langle w_{3|2} - w_{2|3}\rangle = 0$$

$$\bar{u}_2 = \langle u_2\rangle + \varepsilon\langle y_3\rangle\Phi_1$$

$$\bar{u}_3 = \langle u_3\rangle - \varepsilon\langle y_2\rangle\Phi_1$$

$$\bar{u}_1 = \langle u_1\rangle + \varepsilon\langle y_3\rangle\bar{u}_3' + \varepsilon\langle y_2\rangle\bar{u}_2'$$

$$\varepsilon_{11} = \epsilon_1(x_1) + \varepsilon y_3 \kappa_2(x_1) - \varepsilon y_2 \kappa_3(x_1) + \underbrace{w_{1|1}} + \underline{\varepsilon w_{1,1}}$$

$$\varepsilon_{22} = w_{2|2} \quad \varepsilon_{33} = w_{3|3} \quad 2\varepsilon_{23} = w_{2|3} + w_{3|2}$$

$$2\varepsilon_{12} = -\varepsilon y_3 \kappa_1 + w_{1|2} + \underbrace{w_{2|1}} + \underline{\varepsilon w_{2,1}}$$

$$2\varepsilon_{13} = \varepsilon y_2 \kappa_1 + w_{1|3} + \underbrace{w_{3|1}} + \underline{\varepsilon w_{3,1}}$$

MSG-based Multiscale Beam Modeling

- Energy

$$U = \int_0^L \frac{1}{\omega} \left\langle\!\!\left\langle \frac{1}{2} \epsilon^T C \epsilon \right\rangle\!\!\right\rangle \mathrm{d}x_1 \qquad \epsilon = \lfloor \varepsilon_{11} \quad \varepsilon_{22} \quad \varepsilon_{33} \quad 2\varepsilon_{23} \quad 2\varepsilon_{13} \quad 2\varepsilon_{12} \rfloor$$

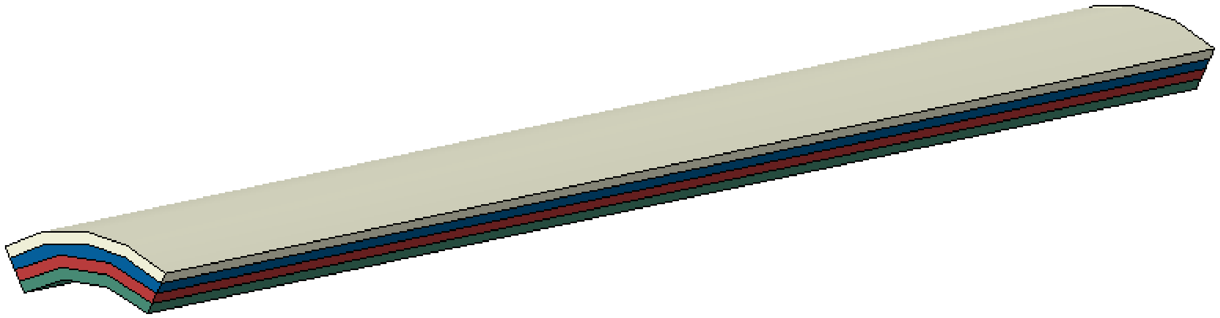
$$W = \int_0^L \frac{1}{\omega} \left(\langle\langle f_i u_i \rangle\rangle + \oint_{\partial\Omega} t_i u_i \, \mathrm{d}s \right) \mathrm{d}x_1 + [t_i u_i]_{x_1=0} + [t_i u_i]_{x_1=L}$$

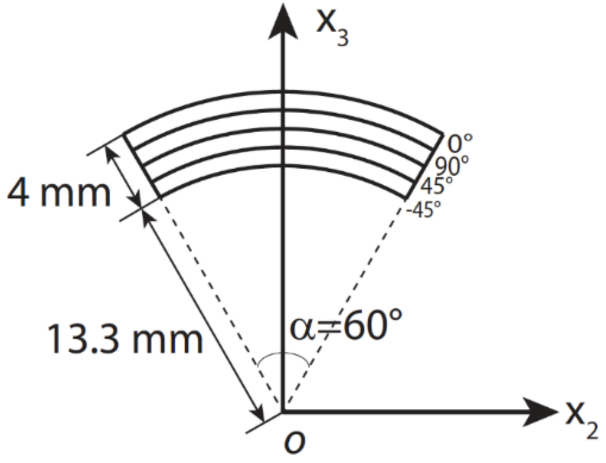
$$\int_0^L \left(\frac{1}{2} \bar{\varepsilon}^T \bar{C} \bar{\varepsilon} - p_i \bar{u}_i - q_i \Phi_i \right) \mathrm{d}x_1 - (P_i \bar{u}_i + Q_i \Phi_i)|_{x_1=0}$$
$$- (P_i \bar{u}_i + Q_i \Phi_i)|_{x_1=L}$$

- Minimize the energy loss to solve for w_i

$$\int_0^L \left(\frac{1}{2} \bar{\varepsilon}^T \bar{C} \bar{\varepsilon} - p_i \bar{u}_i - q_i \Phi_i \right) \mathrm{d}x_1 - (P_i \bar{u}_i + Q_i \Phi_i)|_{x_1=0}$$
$$- (P_i \bar{u}_i + Q_i \Phi_i)|_{x_1=L}$$

Accurate Free-Edge Stress Analysis for a Curved Section





Length:120 mm, length/width≅7

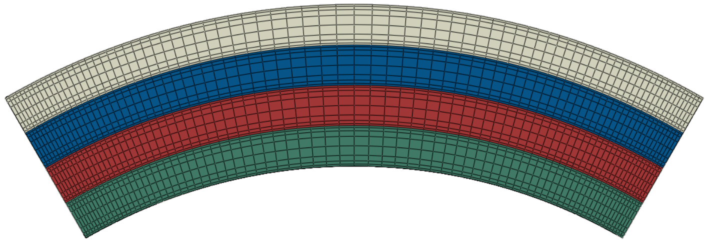
Boundary Conditions

- Case 1: Shear force F_2 = 100 N
- Case 2: Shear force F_3 = 100 N

E_1(MPa)	E_2(MPa)	E_3(MPa)	G_{12}(MPa)	G_{13}(MPa)	G_{23}(MPa)	v_{12}	v_{13}	v_{23}
132000	10800	10800	5650	5650	3380	0.24	0.24	0.59

Computational Cost Comparison

- MSG cross-sectional model

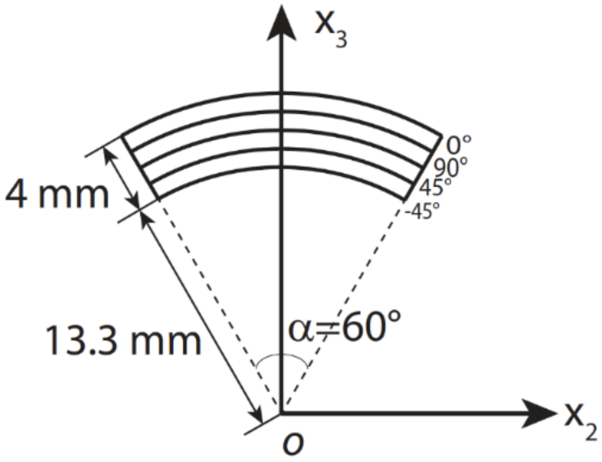


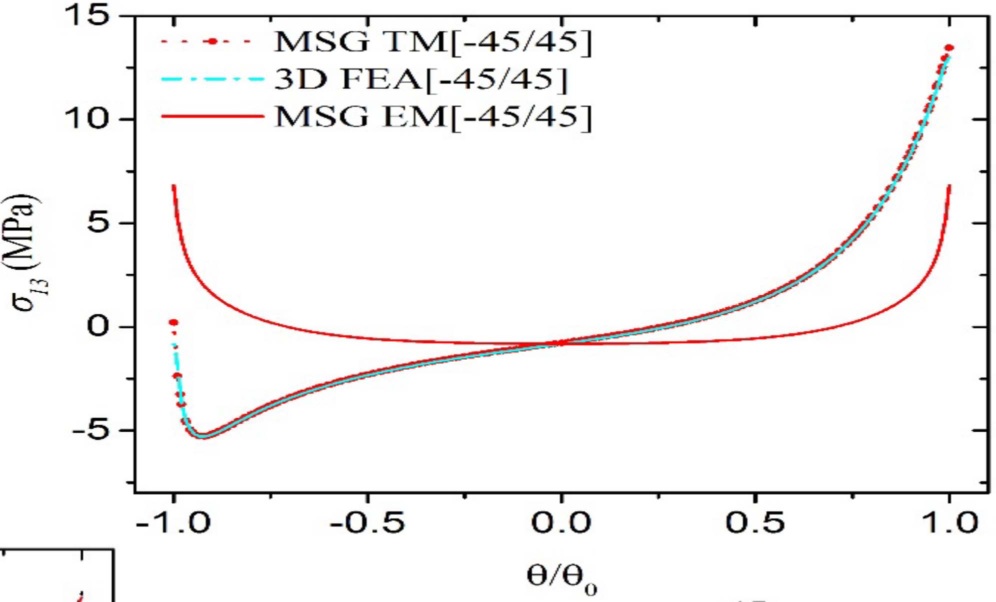
6,937 nodes
2,240 8-noded quads
<3 seconds with 1 CPU

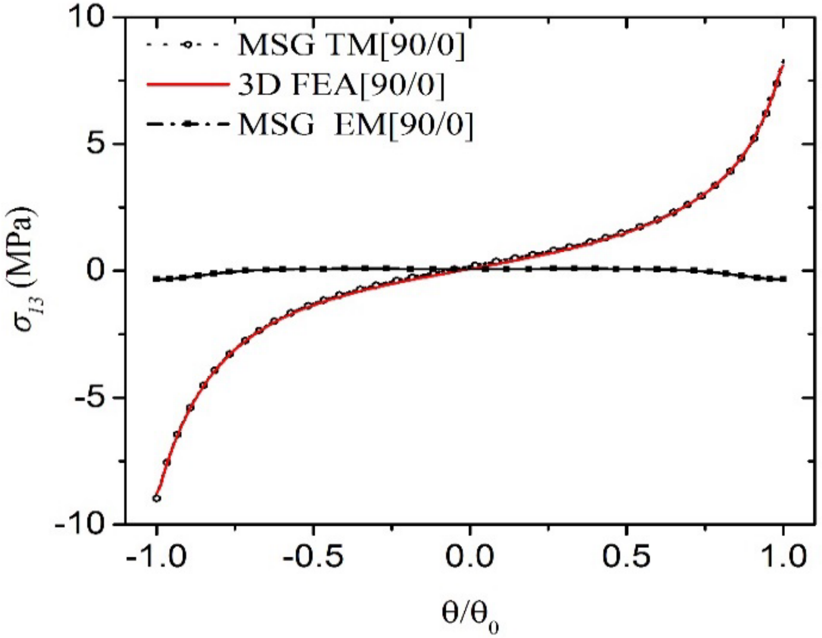
- 3D FEA model:120 mm long, length/width≅7 (4 layers only)

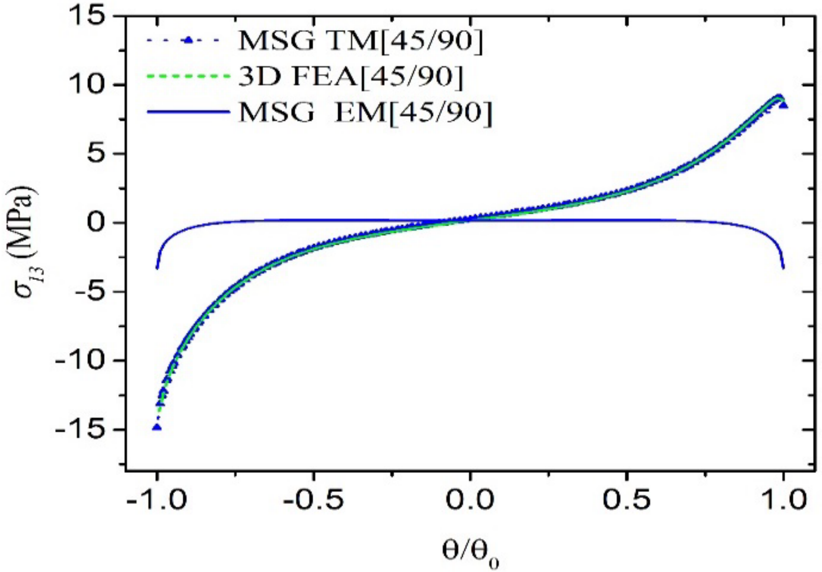
>4M nodes
>1M C3D20Rs
4 hours with 24 CPUs
Prohibitive for more realistic structures, e.g. flexbeam (100+ layers)

Inter-laminar Shear Stresses under F_2

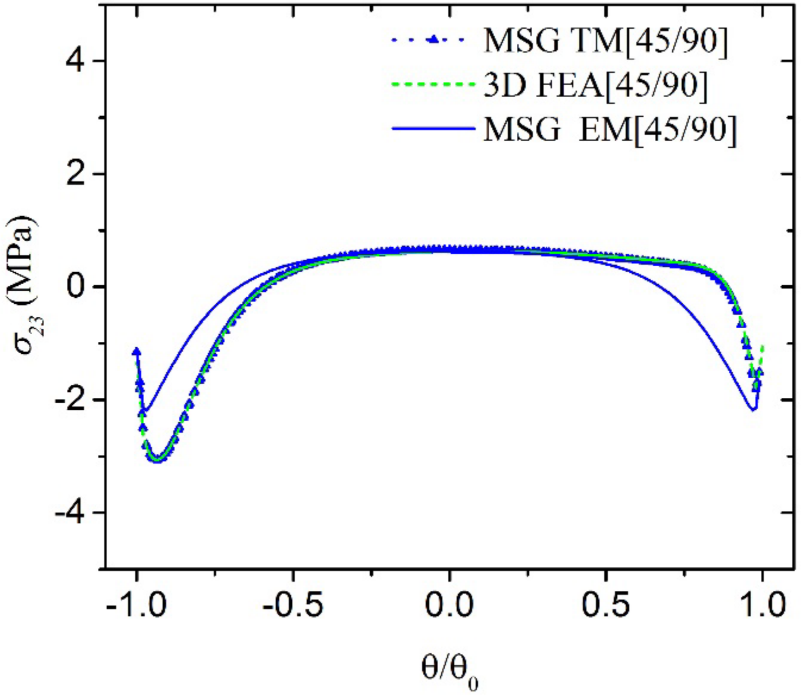


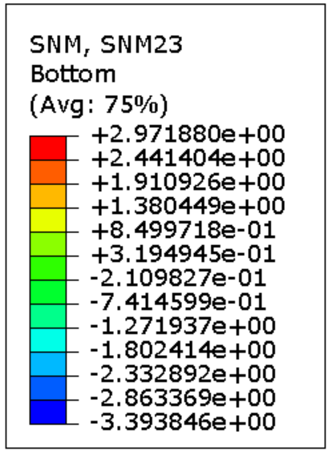


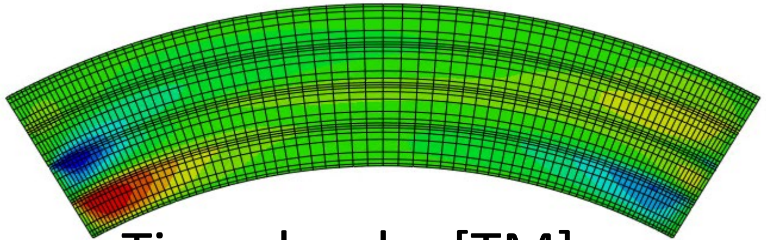




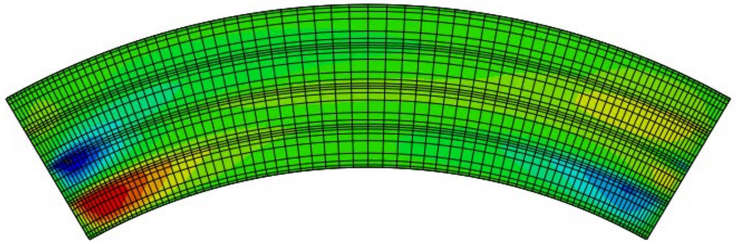
Inter-laminar Shear Stresses under F_2



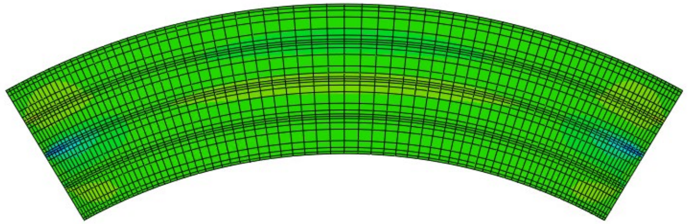




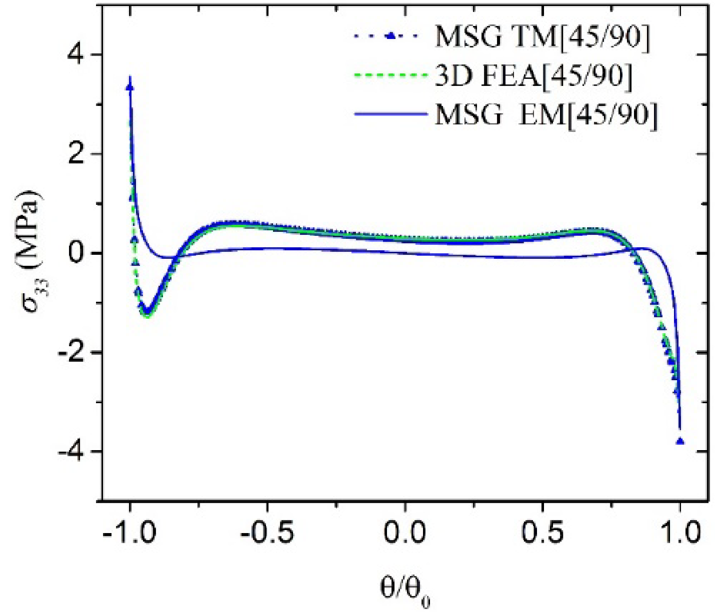
Timoshenko [TM]



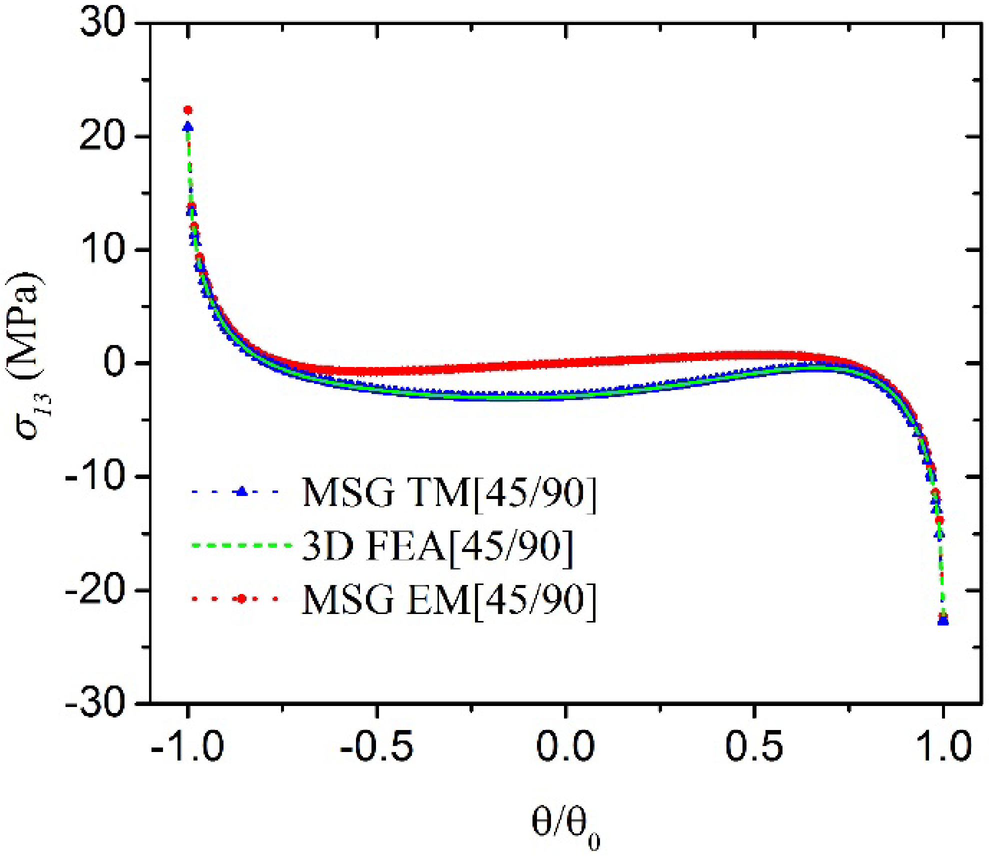
3D FEA

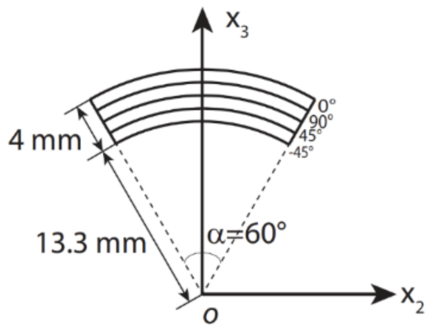


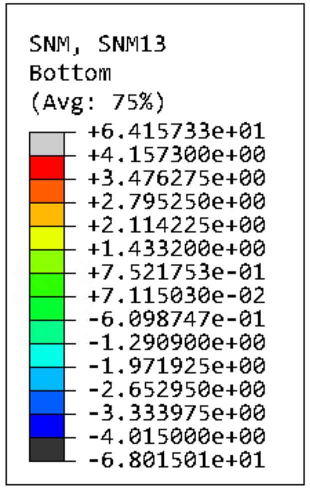
Euler [EM]

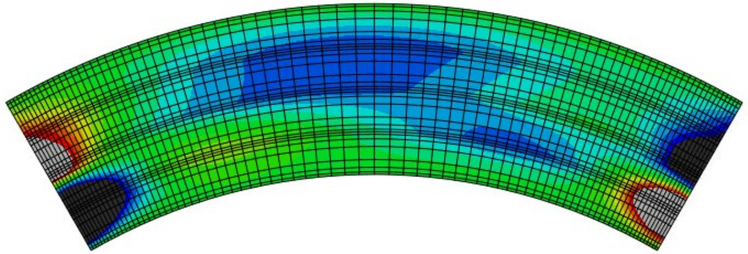


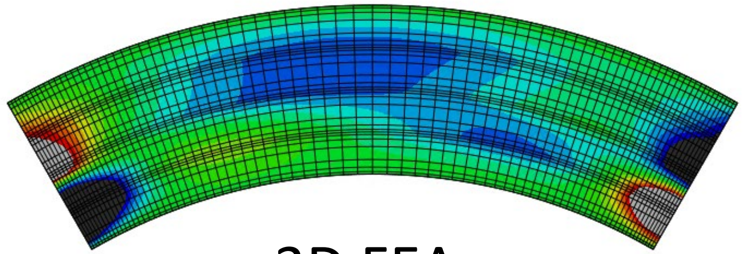
Inter-laminar Shear Stresses under F_3

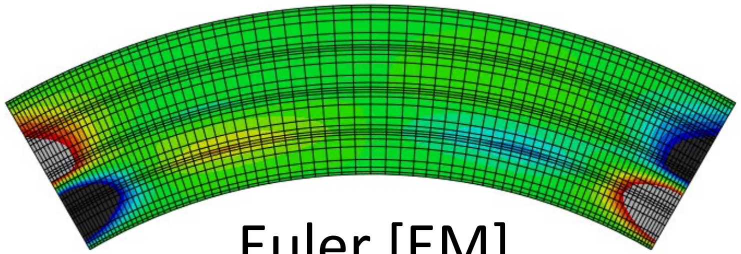







Timoshenko [TM]


3D FEA


Euler [EM]

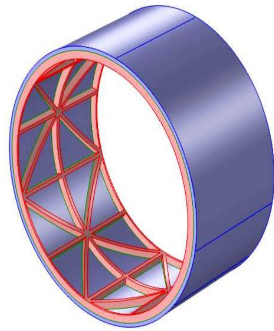
Stiffened Composite Cylinder

Skin layup: [45/−45/90/0/45]s, thickness 0.09 in
Stiffener width & depth: 0.18 in

unit (psi)

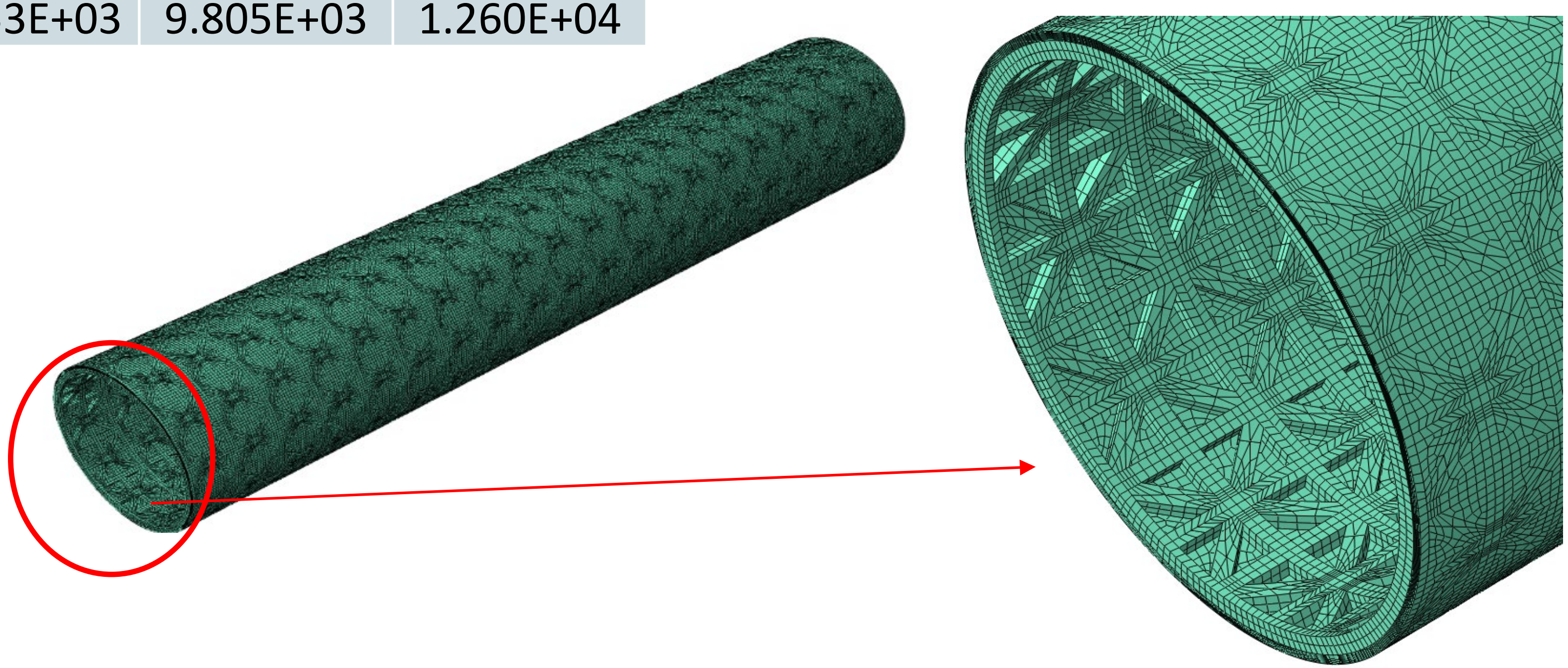
E_1	$E_2=E_3$	$G_{12}=G_{13}$	G_{23}	$\nu_{12}=\nu_{13}$	ν_{23}
1.923E+07	1.566E+06	8.267E+05	4.931E+05	0.24	0.49

X^+	$Y^+=Z^+$	X^-	$Y^-=Z^-$	R	S=T
2.205E+05	6.353E+03	2.466E+05	6.353E+03	9.805E+03	1.260E+04

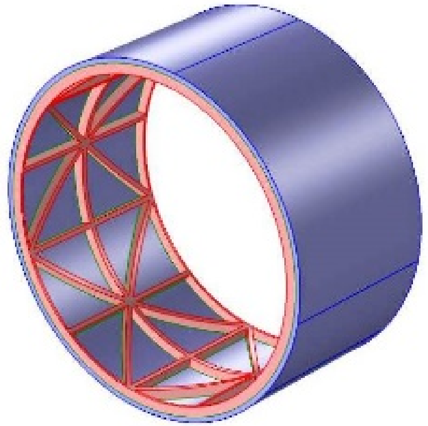


SG

Cylinder has 20 SGs



Stiffened Composite Cylinder



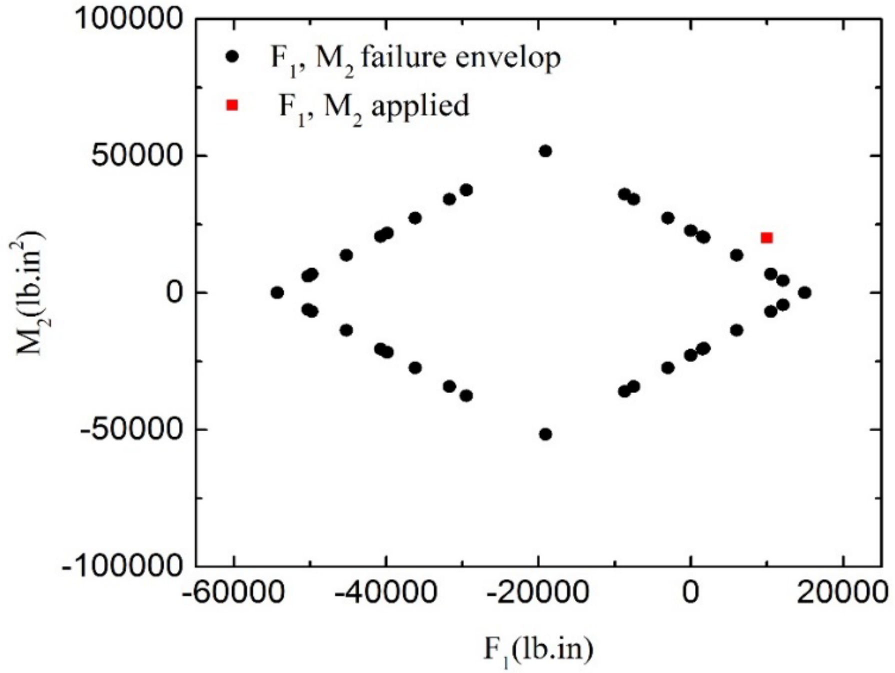
Effective stiffness (Timoshenko model)

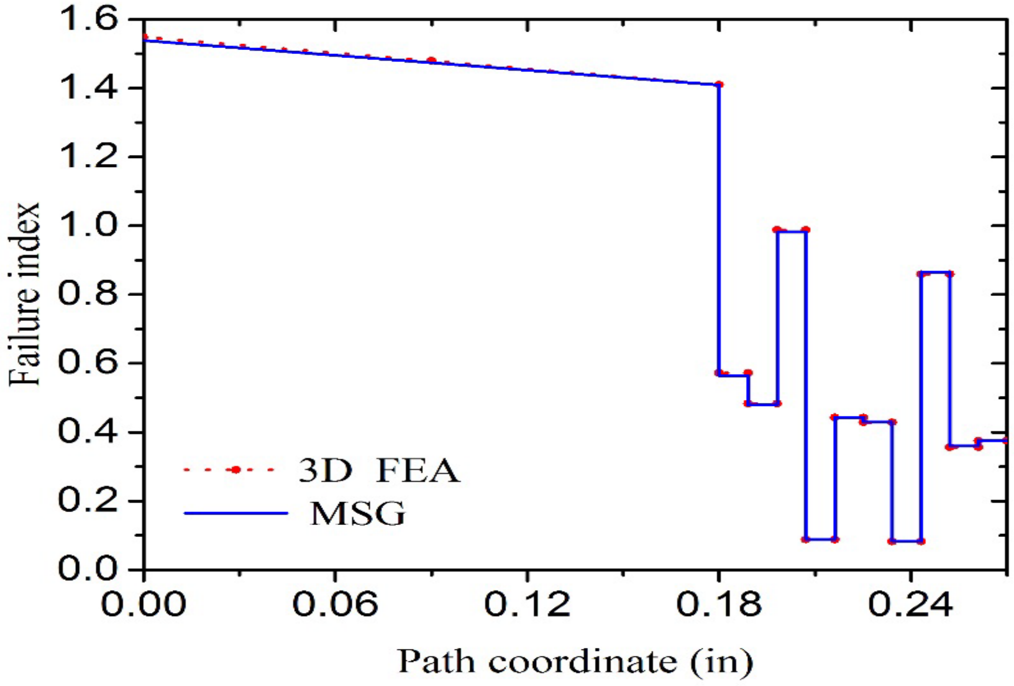
Extension (lb)	Shear (lb)	Twisting (lb•in^2)	Bending (lb•in^2)	Shear-bending (lb•in)	Extension-twisting (lb•in)
1.192E7	2.153E6	3.763E7	5.131E7	3.981E5	-8.143E5

Effective strength (Timoshenko model)

Direction	F_1 (lb)	F_2=F_3 (lb)	M_1 (lb•in)	M_2=M_3 (lb•in)
+	1.498E4	8.782E3	5.144E4	2.281E4
-	5.430E4	8.782E3	7.073E4	2.281E4

Failure Envelope & Strength Ratio

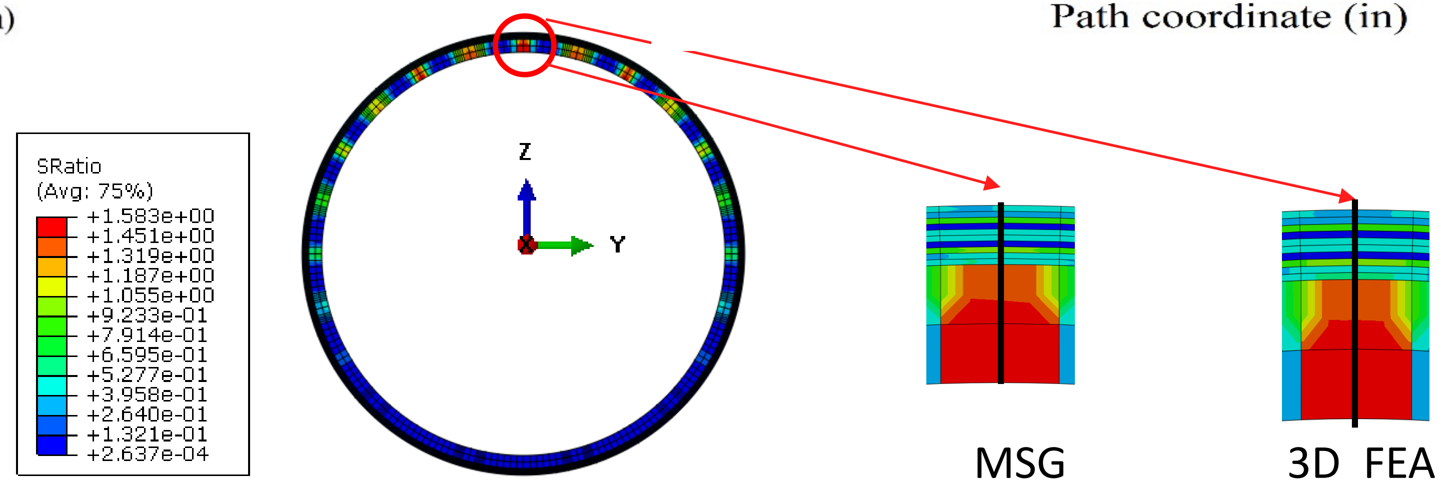




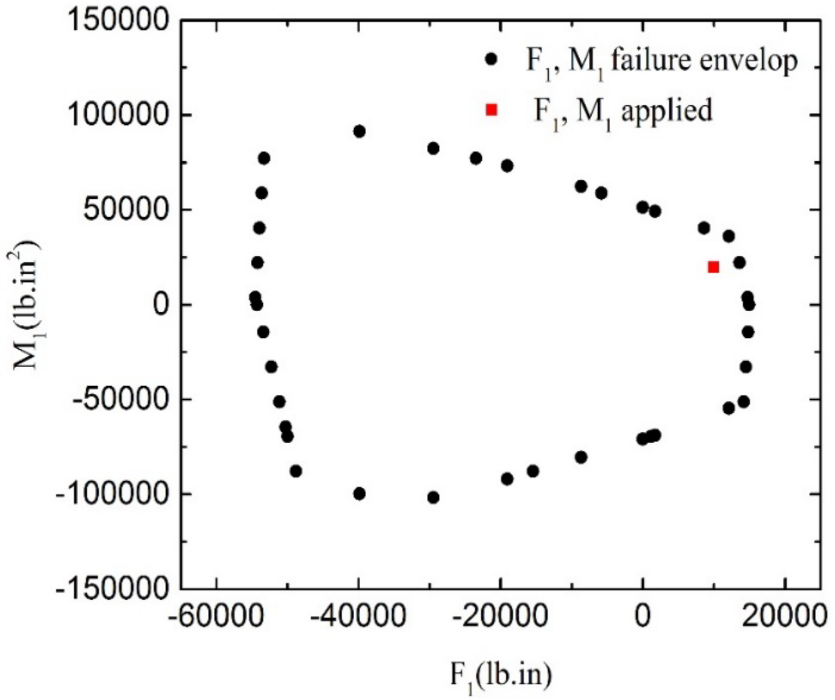
Load:

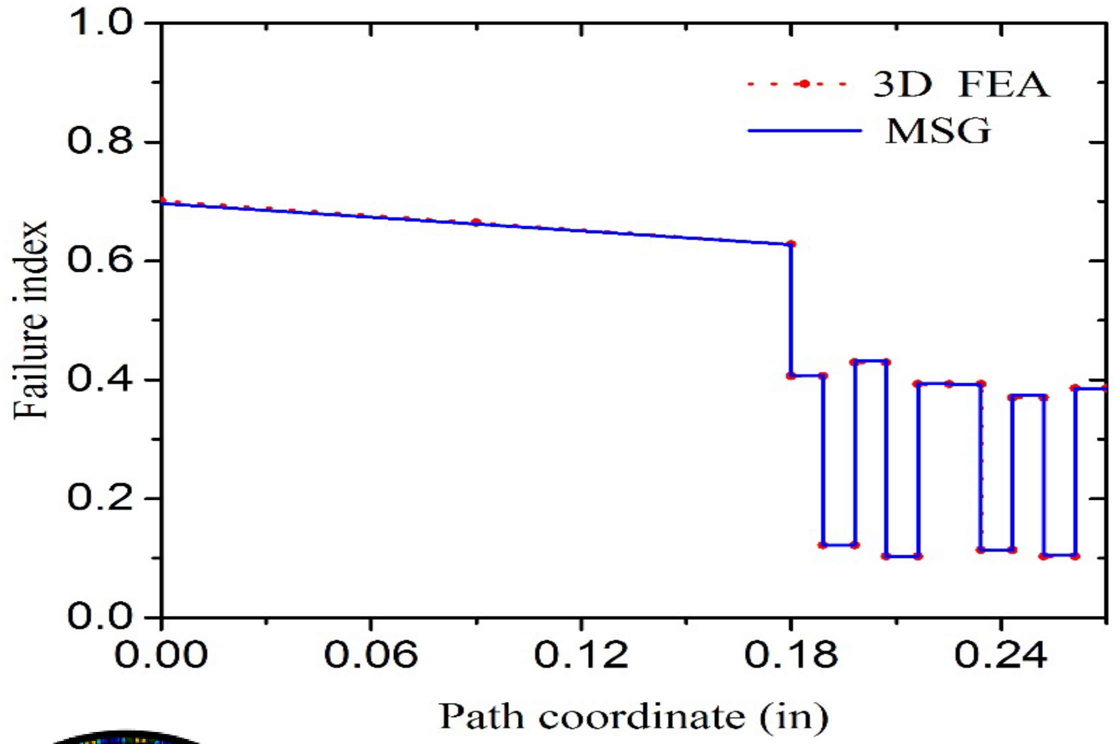
$F_1=10^4$ lb

$M_2=2\text{x}10^4$ lb•in



Failure Envelope & Strength Ratio

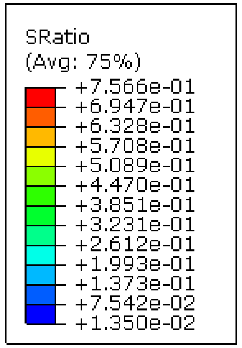


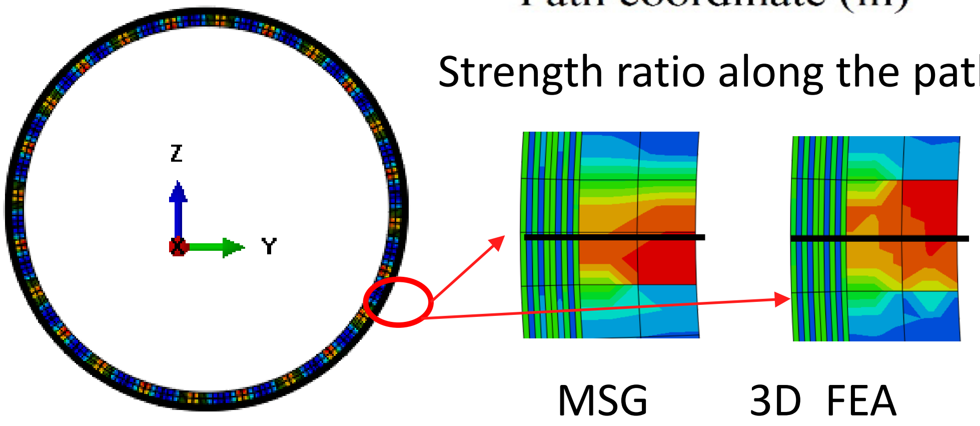


Load:

$F_1 = 10^4$ lb

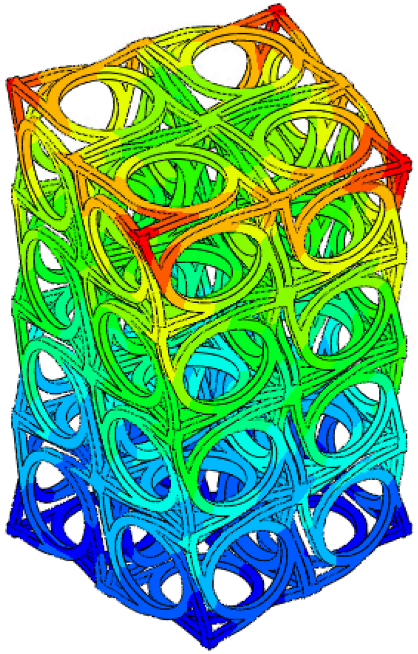
$M_1 = 2\text{x}10^4$ lb•in



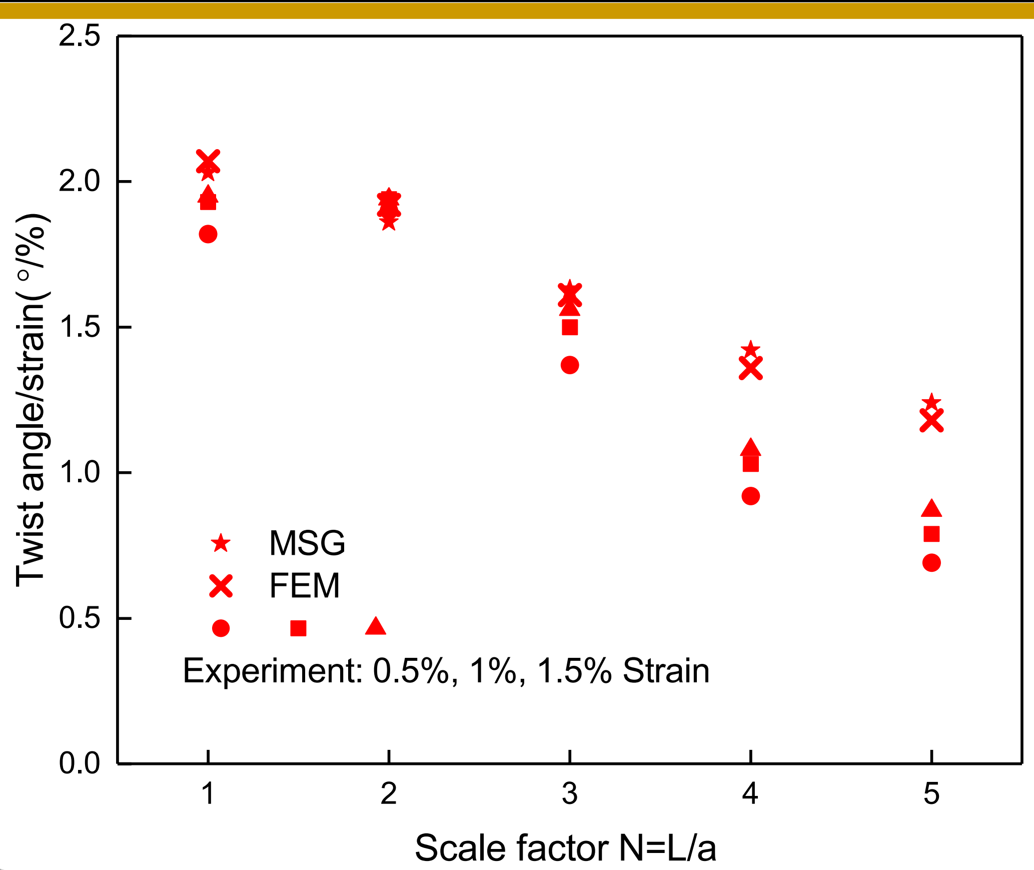


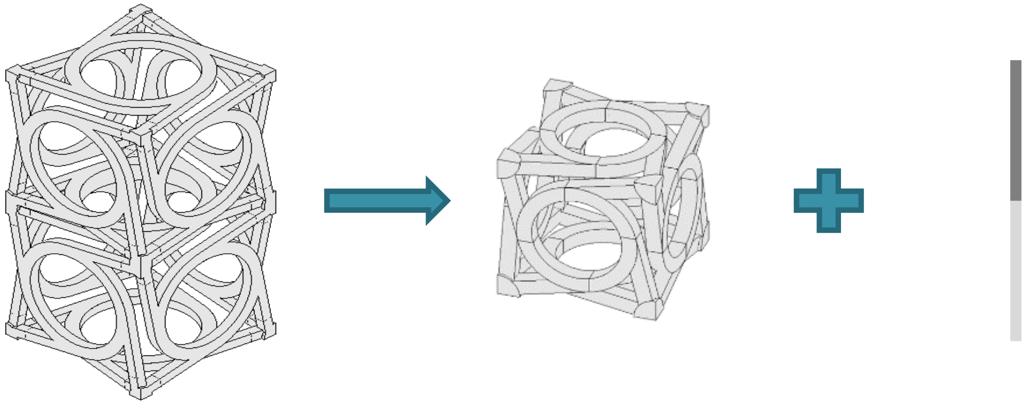
Strength ratio along the path

Constitutive Modeling of Metamaterials



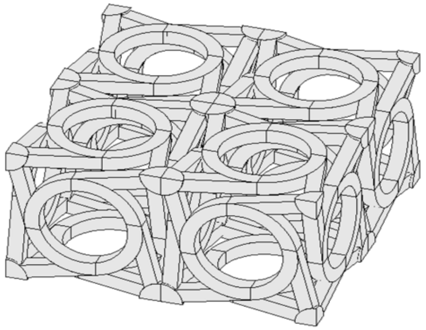
Frenzel, T., Kadic, M., & Wegener, M. (2017). Three-dimensional mechanical metamaterials with a twist. *Science*, *358*, 1072-1074.



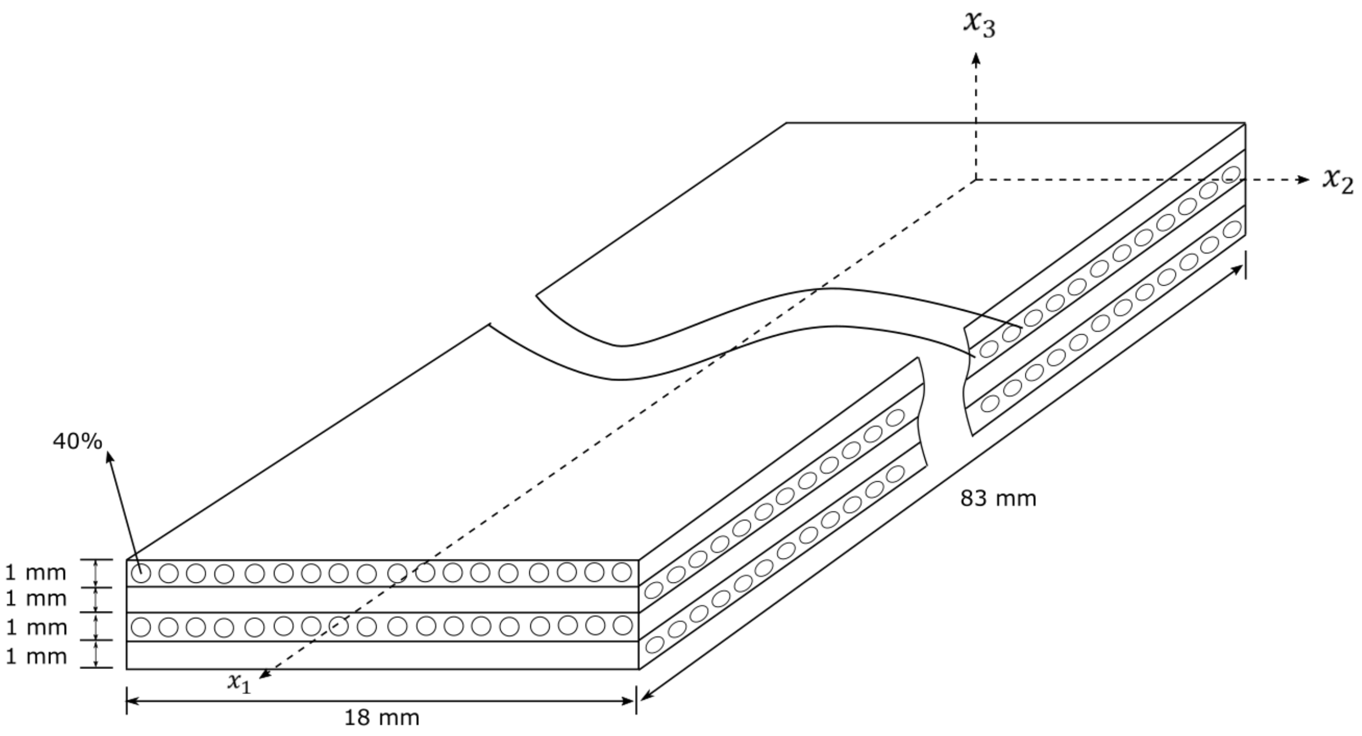


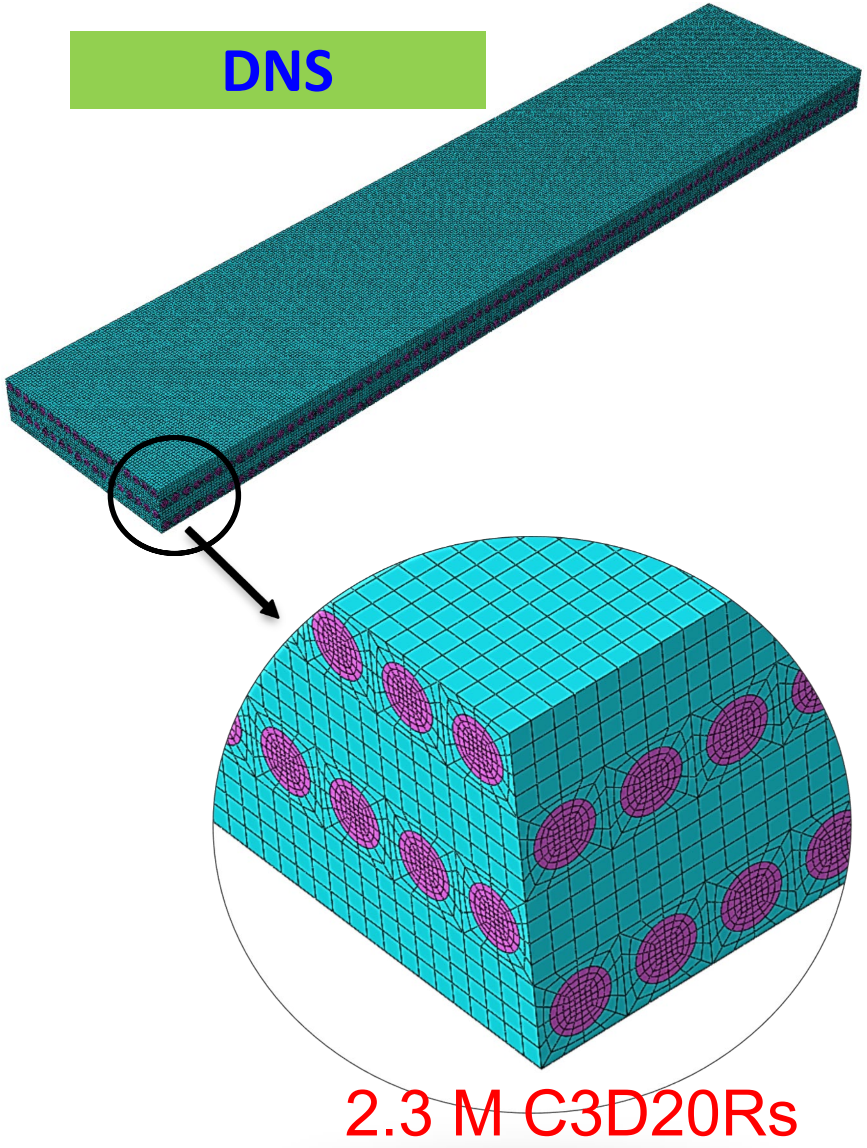
3D SG

Beam



Four Layer Cross-Ply Laminate

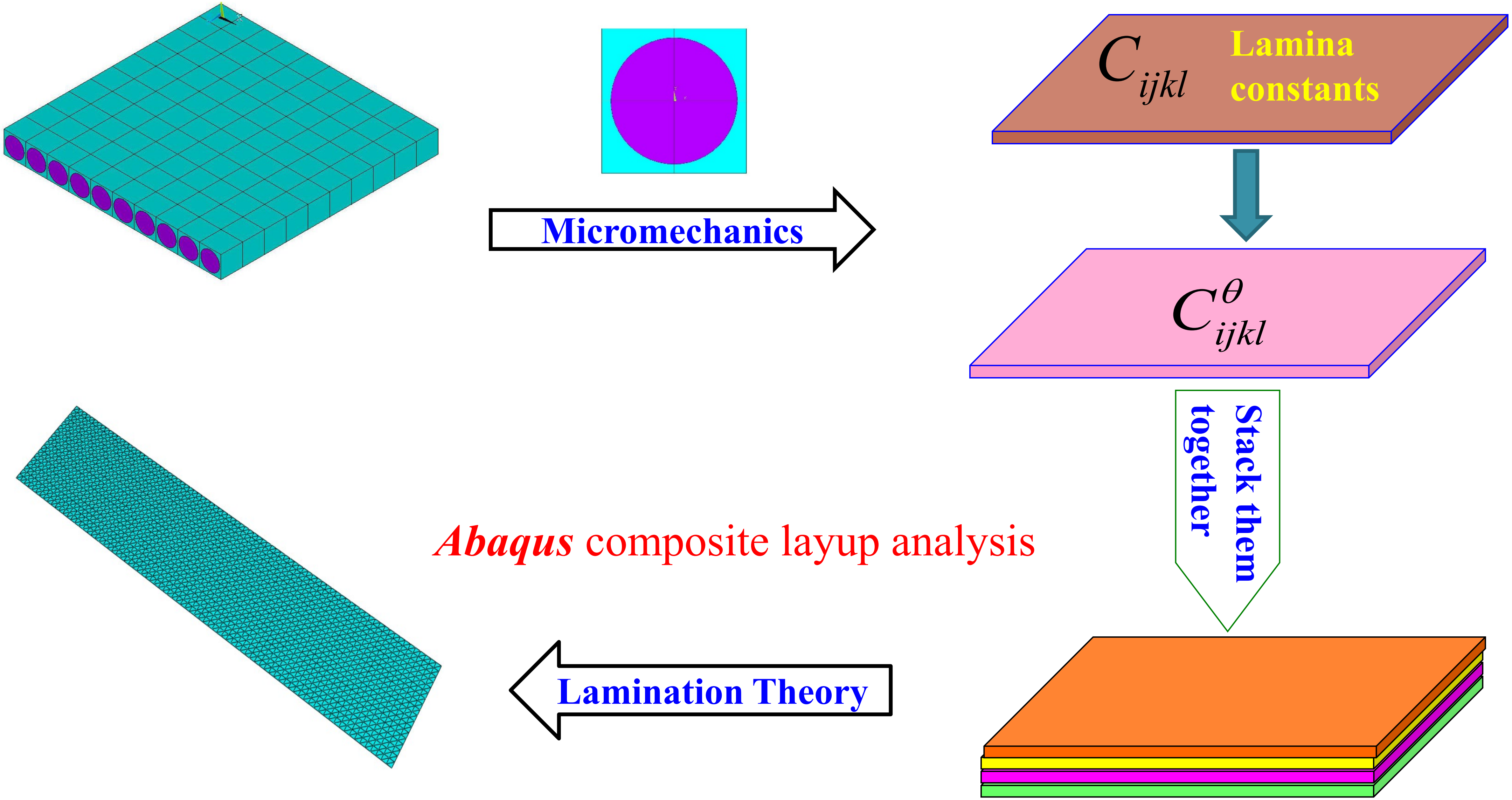




Cantilever with a tensile load at the geometry center of the tip section

Material	E (GPa)	ν
Fiber	276	0.28
Matrix	4.76	0.37

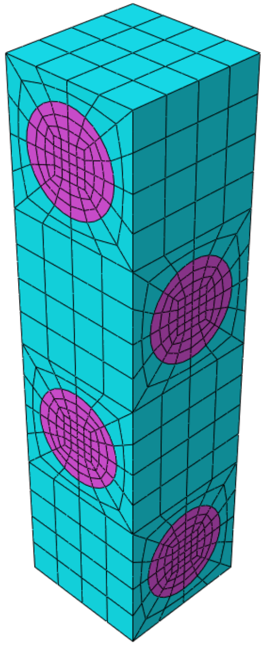
Bottom-up Multiscale Modeling



MSG-based Multiscale Modeling

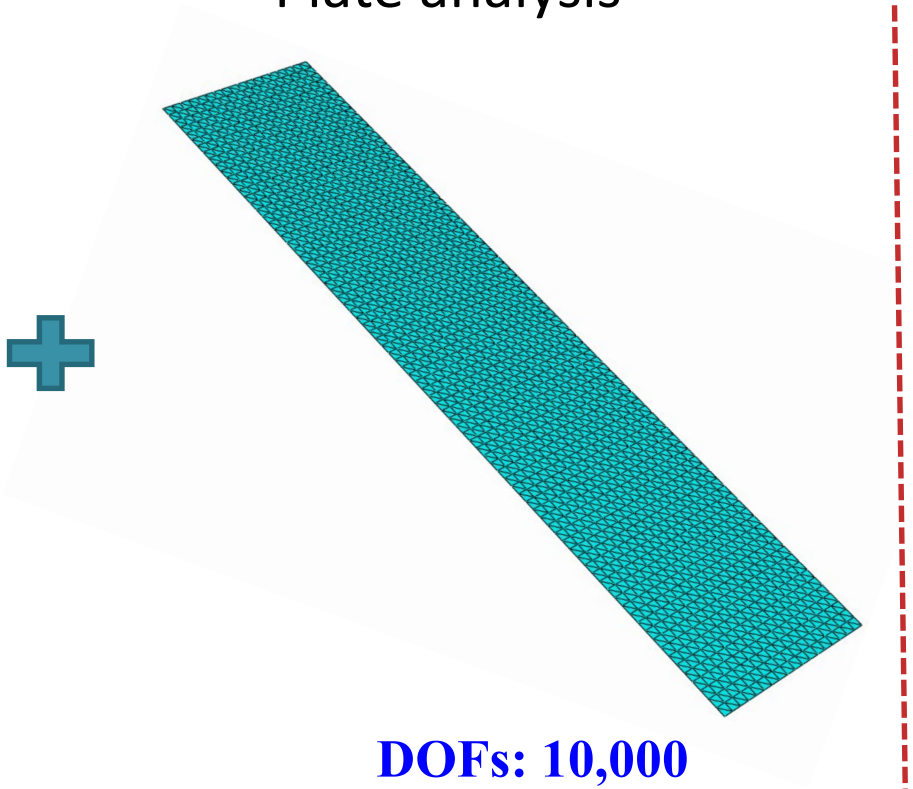
MSG-based Plate Analysis

Plate SG



DOFs: 23,000

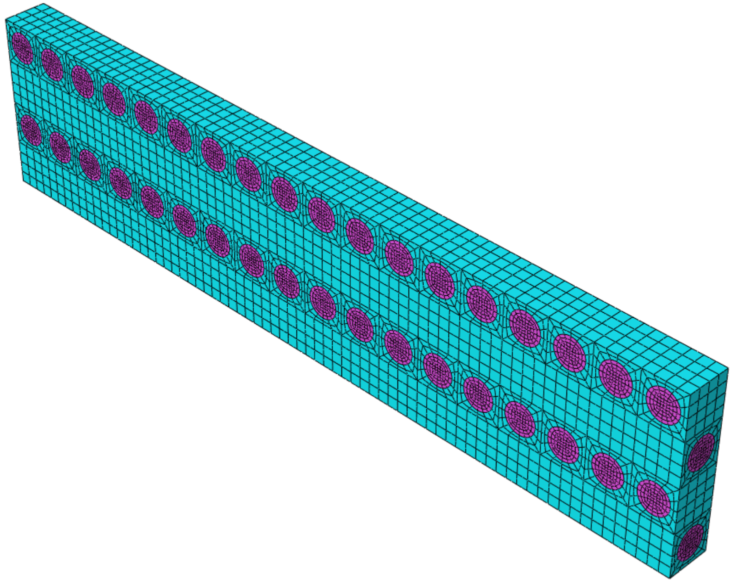
Plate analysis



DOFs: 10,000

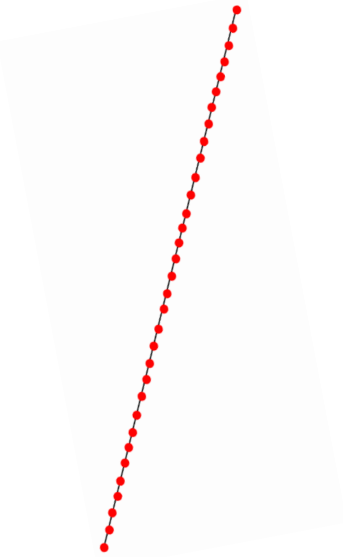
MSG-based Beam Analysis

Beam SG



DOFs: 373,000

Beam analysis

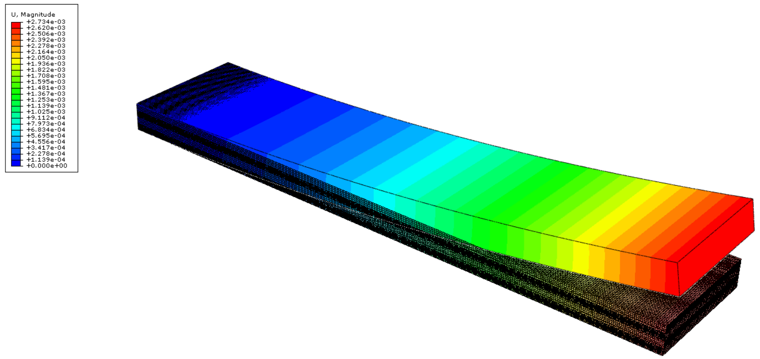


DOFs: 500

Global Behavior

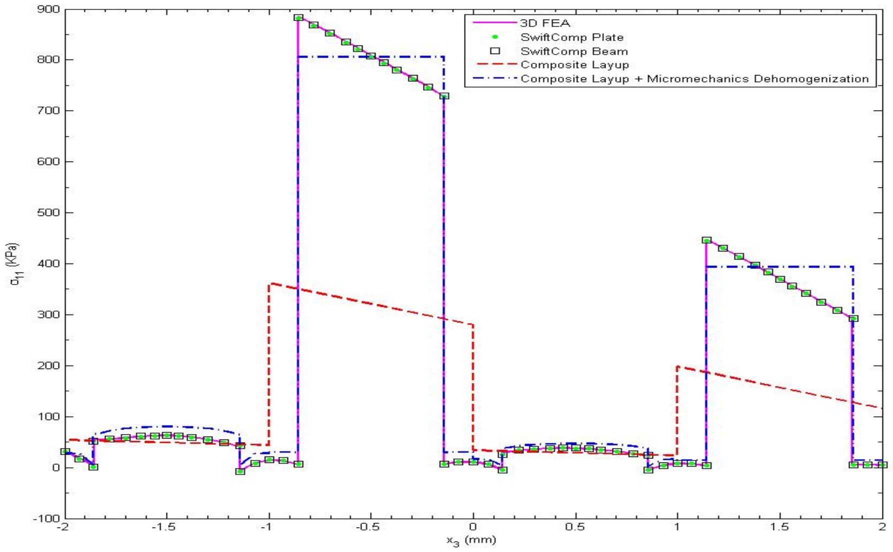
Method	U_1	Absolute error
3D FEA	2.0849×10^{-4}	
MSG Beam	2.0873×10^{-4}	0.1151%
MSG Plate	2.0832×10^{-4}	0.0815%
ABAQUS Composite layup	2.0804×10^{-4}	0.2158%

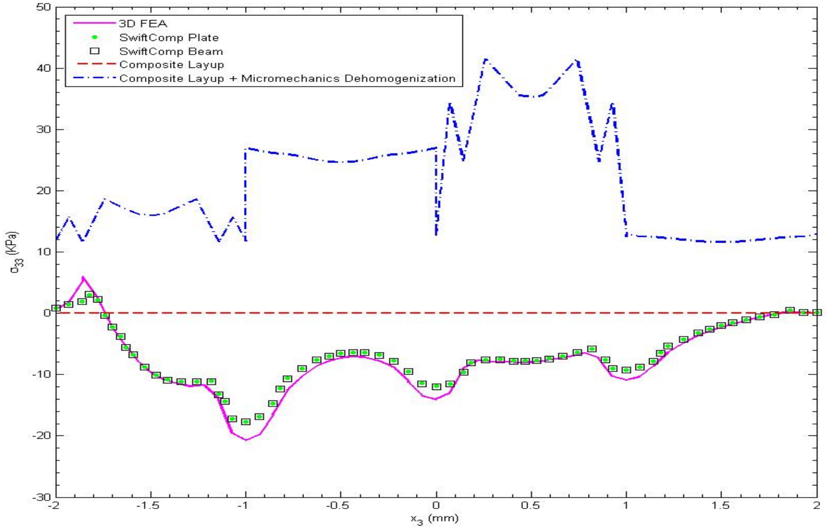
Method	U_3	Absolute error
3D FEA	2.7124×10^{-3}	
MSG beam	2.7146×10^{-3}	0.0811%
MSG plate	2.7084×10^{-3}	0.1475%
ABAQUS Composite layup	2.5264×10^{-3}	**6.8574%**



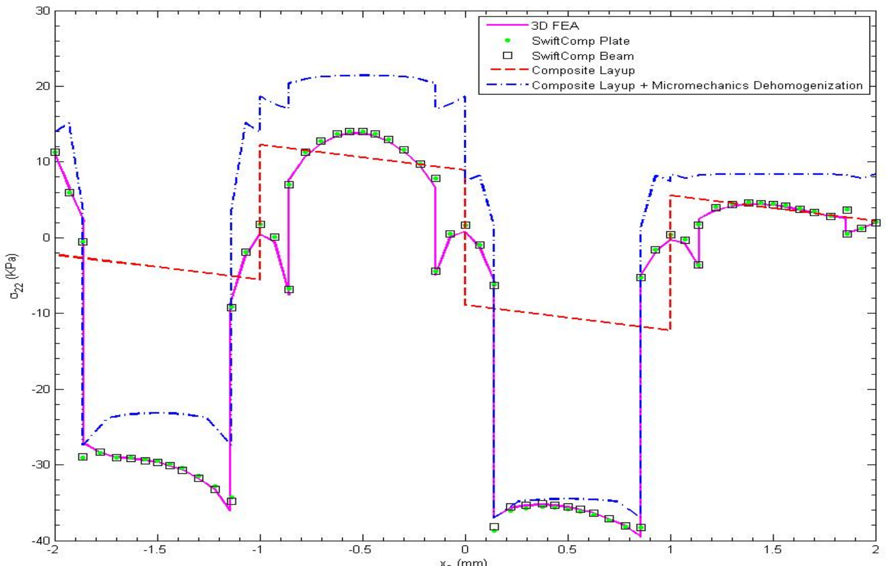
- **Conventional method under predicts the deflection**
- **SwiftComp-based beam and plate analyses both agrees with 3D FEA**

Local Stress Distribution





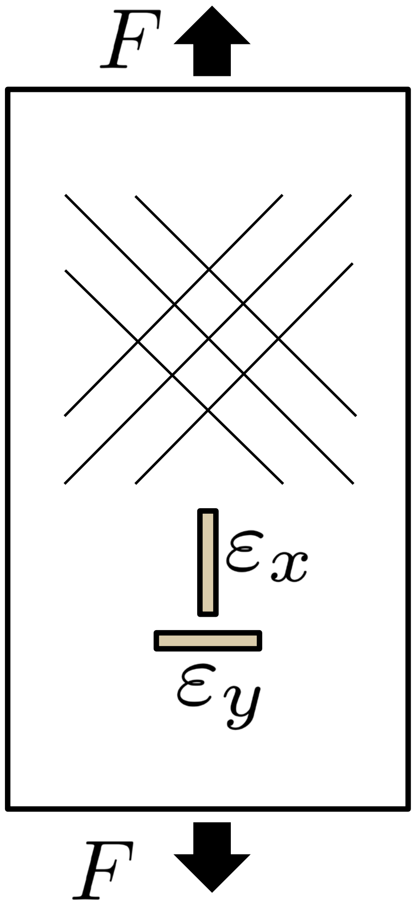
- Conventional method predicts poorly
- MSG-based beam & plate analyses achieve excellent agreements with DNS



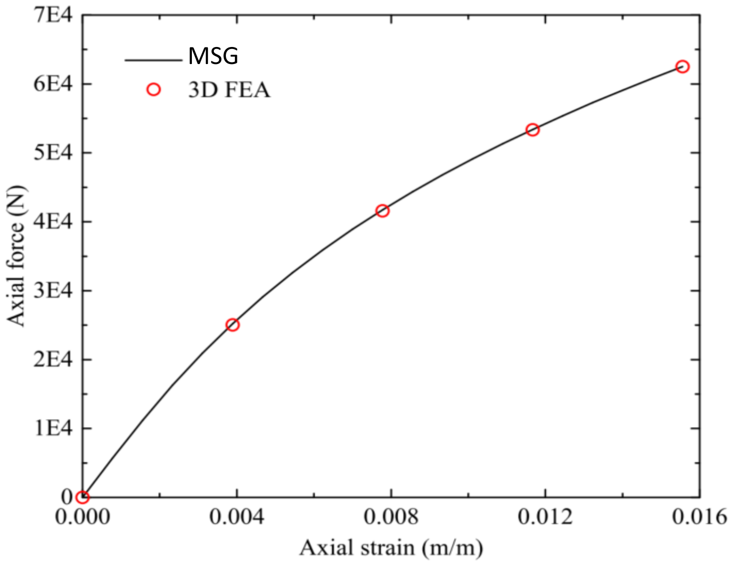
Method	CPUs	Time
DNS	48	7.5 Days
Abaqus composite layup	1	30”
SwiftComp plate analysis	1	40”
SwiftComp beam analysis	1	4’35”

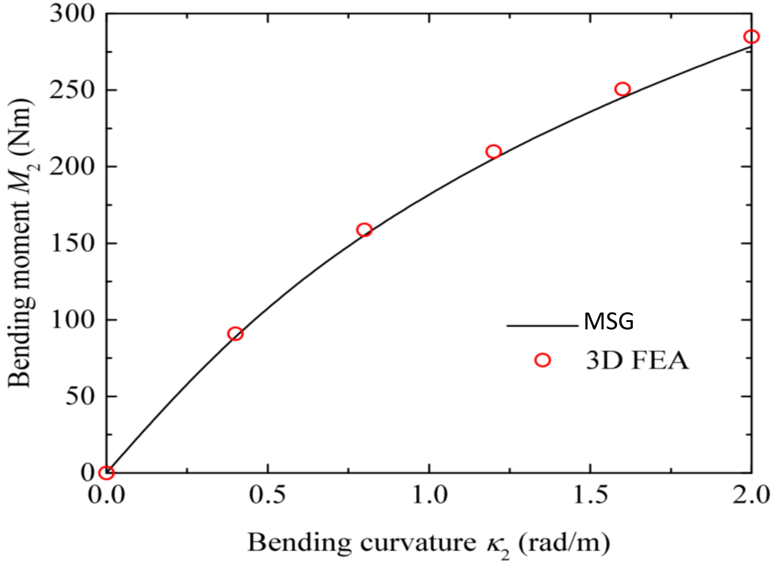
MSG reproduces DNS with $1/10^6$ computing time, as fast as traditional multiscale modeling

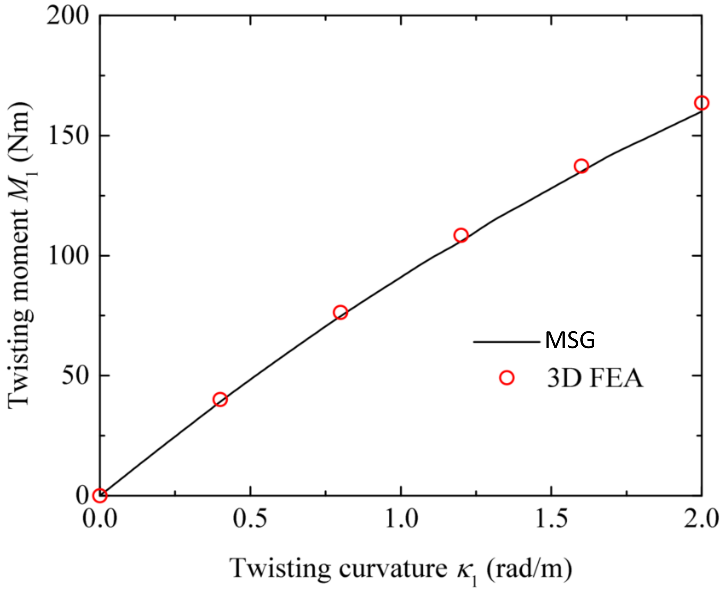
Modeling Nonlinear Shear Behavior



$[\pm 45]$ **laminates**

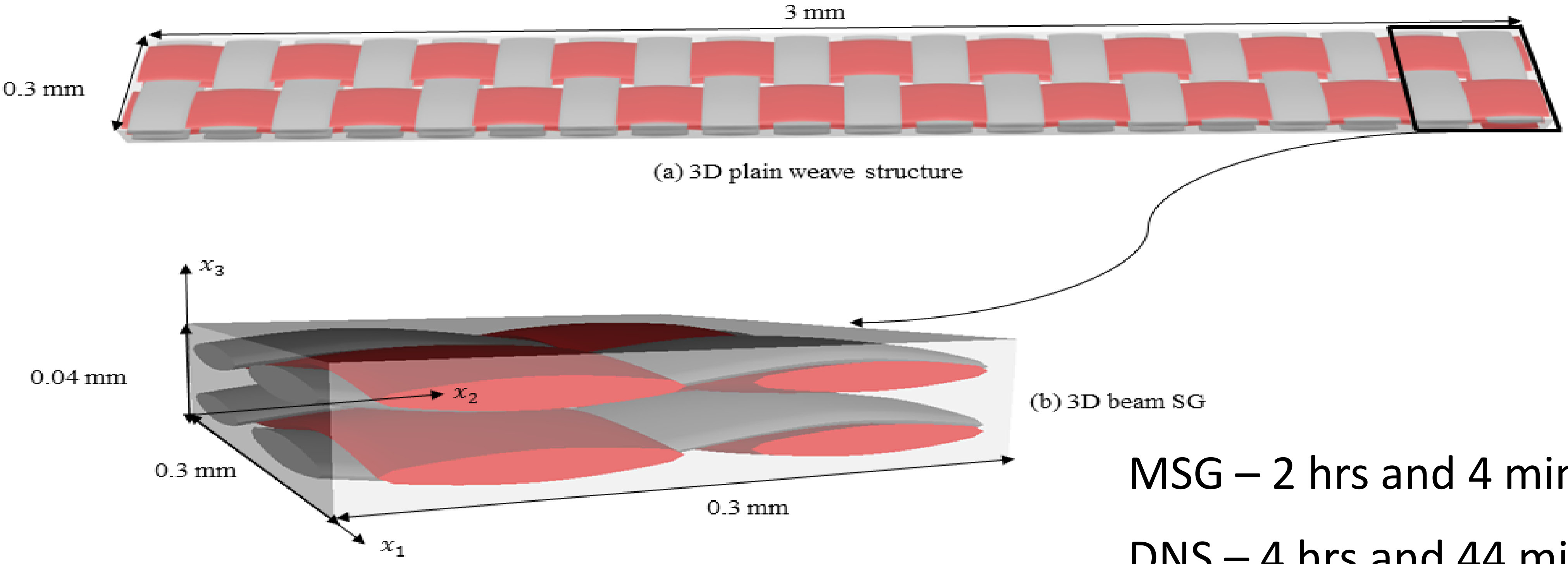






Can be used to calibrated in-plane nonlinear shear behavior using the tensile load-displacement curve then use this calibrated constitutive relations to predict other nonlinear behavior.

MSG Multiscale Structural Modeling



MSG – 2 hrs and 4 mins (1 CPU)

DNS – 4 hrs and 44 mins (28 CPUs)

Boundary conditions: Fixed-free boundary conditions.

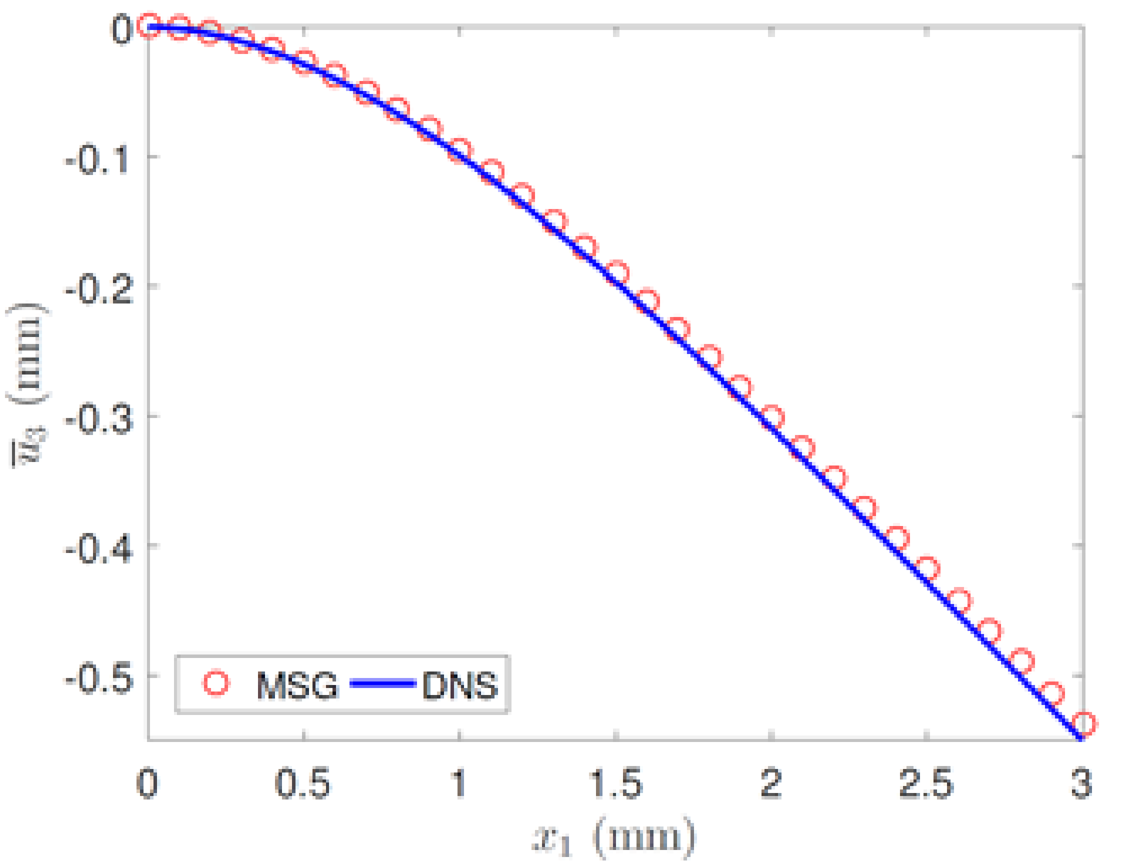
Loading: Uniform pressure in - x_3.

Mesh: (1) SG: 86.4K 20-noded brick elements

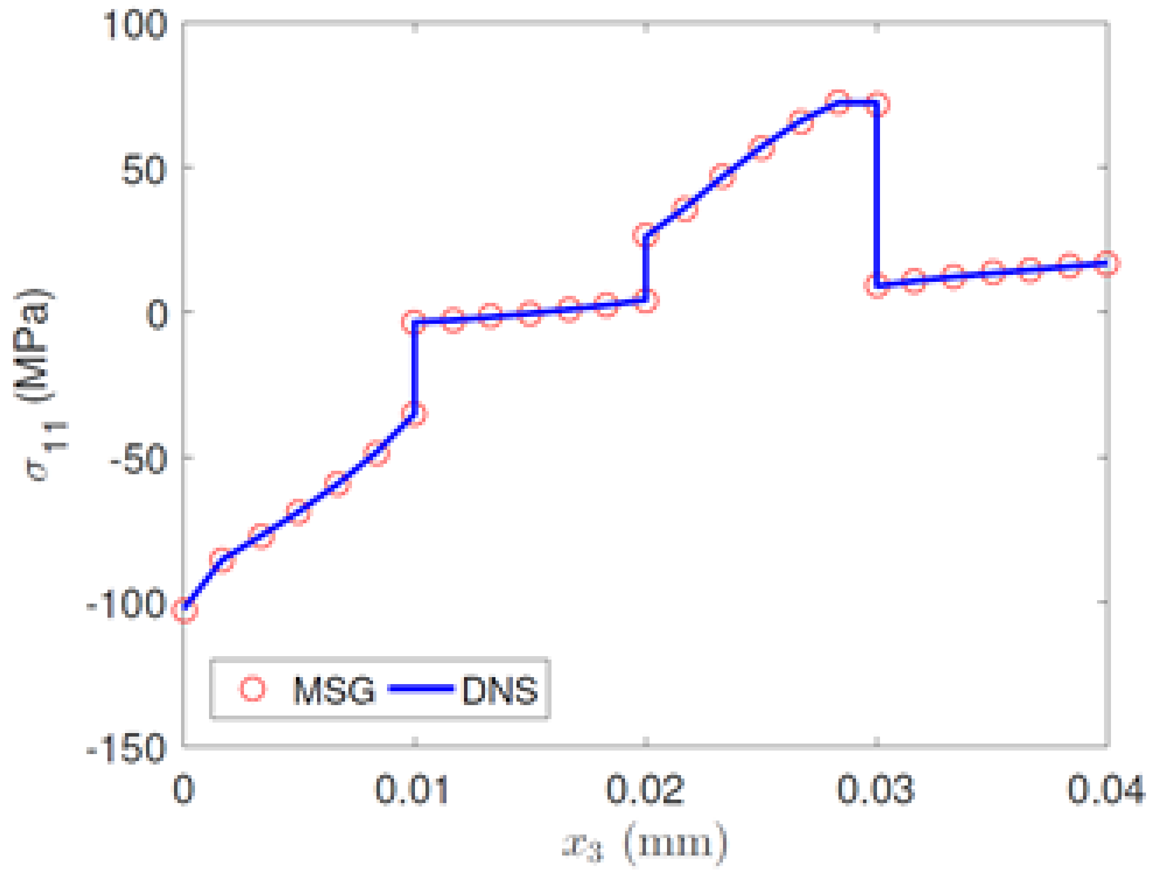
(2) DNS: 864K 20-noded brick elements

$$S = \begin{bmatrix} 4.49\times10^{2}\,\mathrm{N} & 0 & 0 & 2.52\times10^{-2}\,\mathrm{N\cdot mm} \\ 0 & 1.58\times10^{-2}\,\mathrm{N\cdot mm^2} & 0 & 0 \\ 0 & 0 & 5.64\times10^{-2}\,\mathrm{N\cdot mm^2} & 0 \\ 2.52\times10^{-2}\,\mathrm{N\cdot mm} & 0 & 0 & 2.96\,\mathrm{N\cdot mm^2} \end{bmatrix}$$

MSG Multiscale Structural Modeling

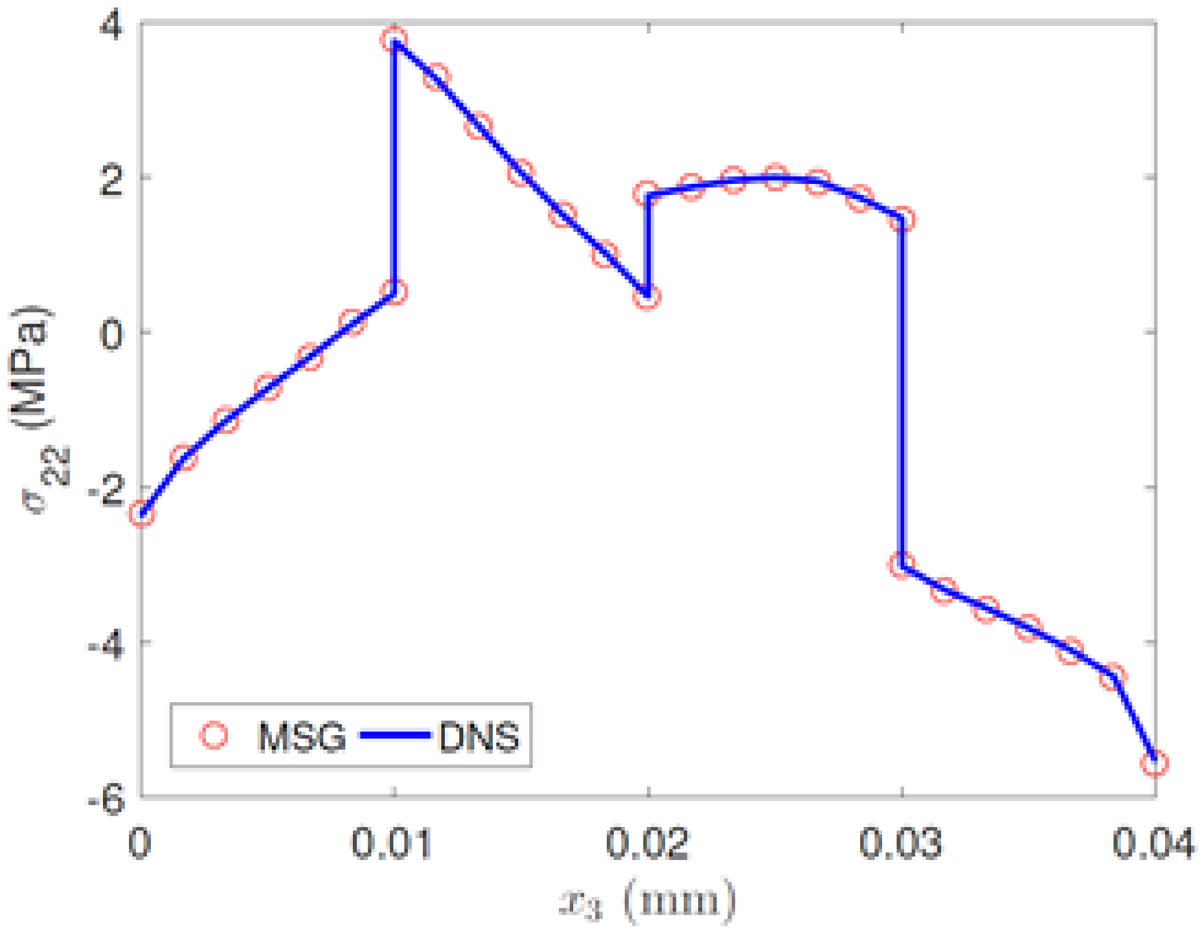


Deflection in plain weave beam along x_1 direction.

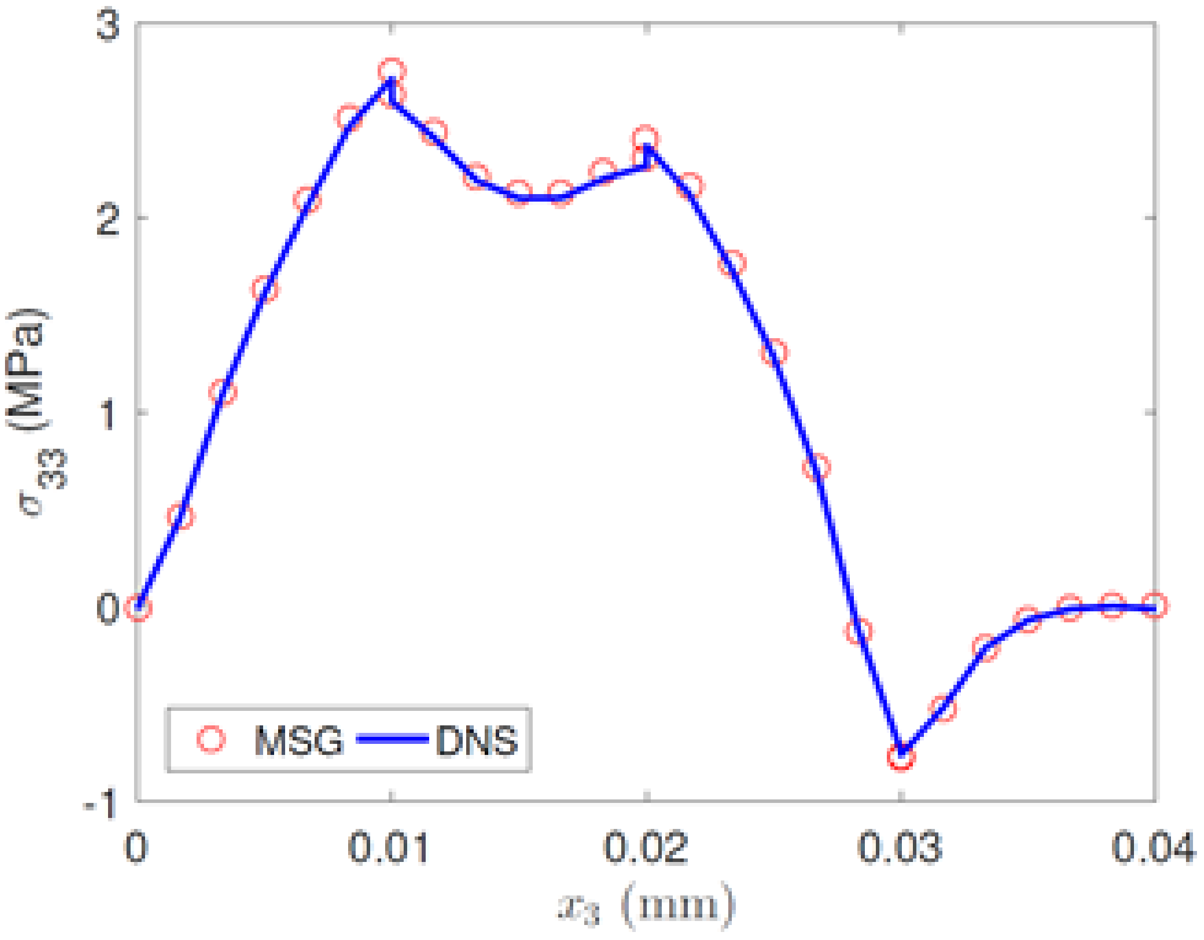


Distribution of σ_{11} through the thickness.

MSG Multiscale Structural Modeling

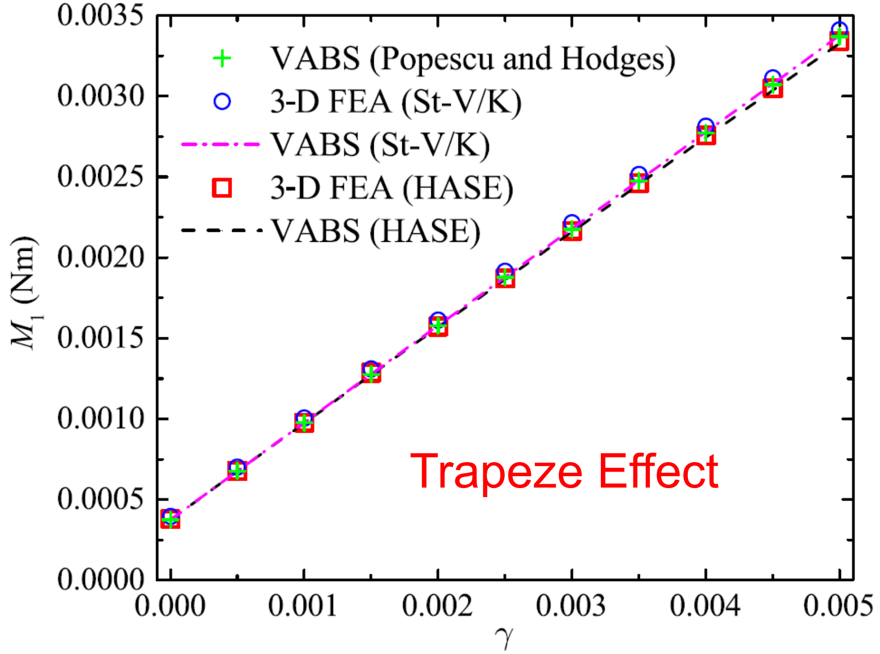


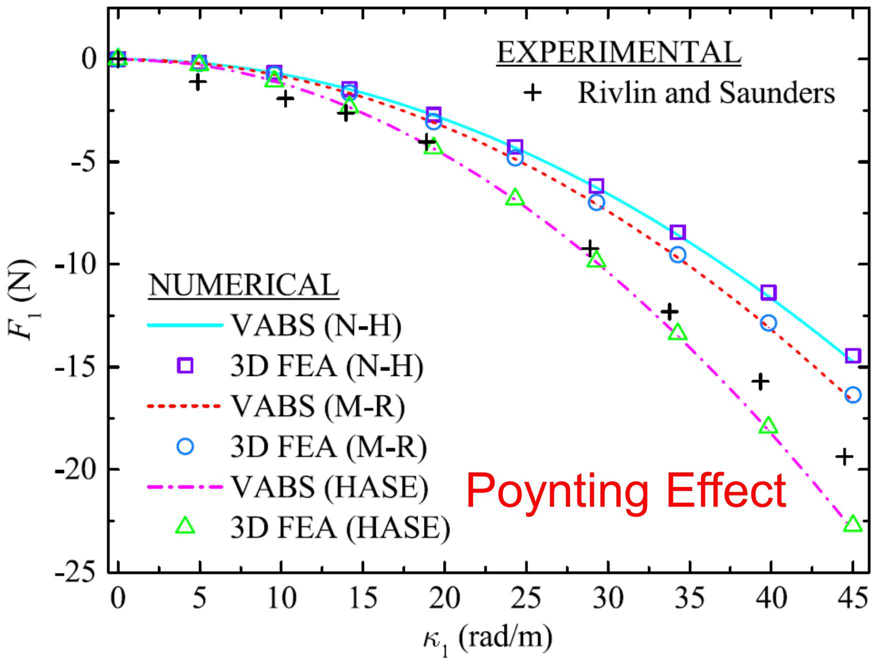
Distribution of σ_{22} through the thickness.

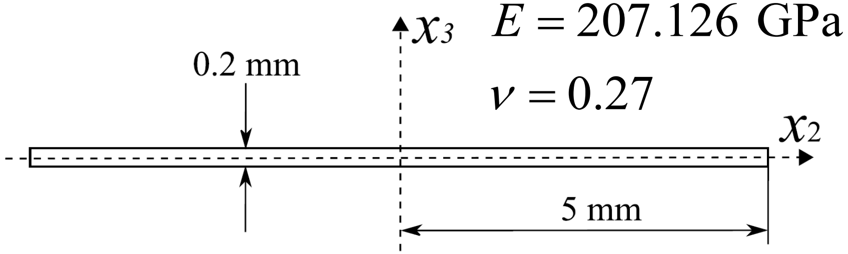


Distribution of σ_{33} through the thickness.

Finite Strain: Trapeze and Poynting Effects





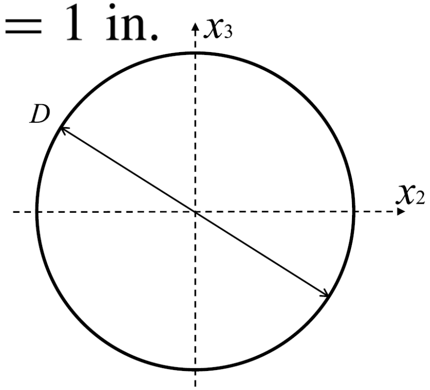


$E = 207.126$ GPa

$\nu = 0.27$

Sectional geometry of spring steel strip.

$D = 1$ in.



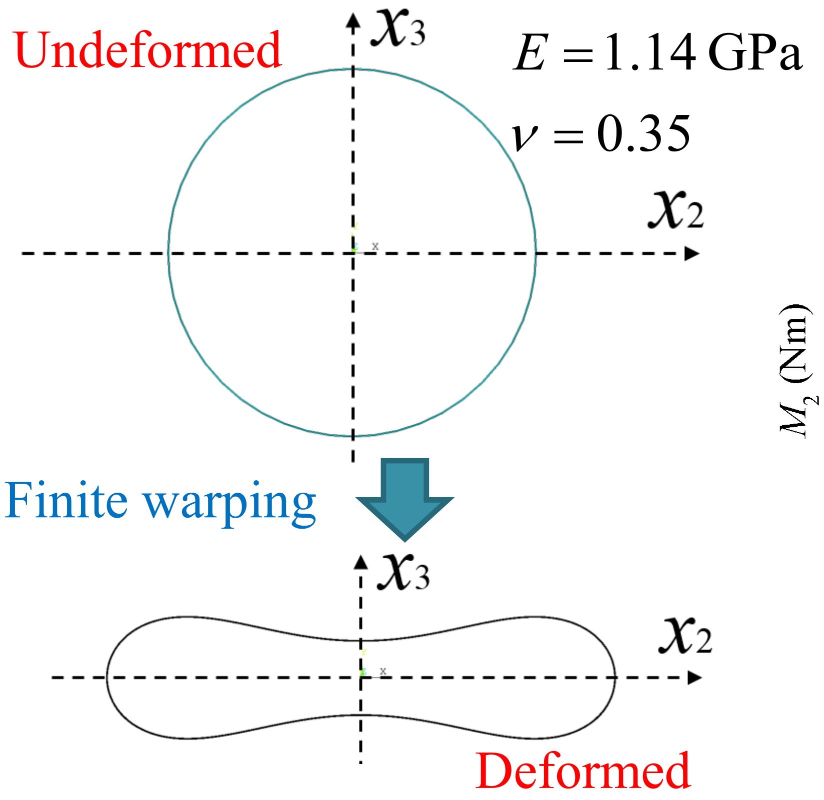
Vulcanized rubber

$G = 382.5\,\text{kPa}$

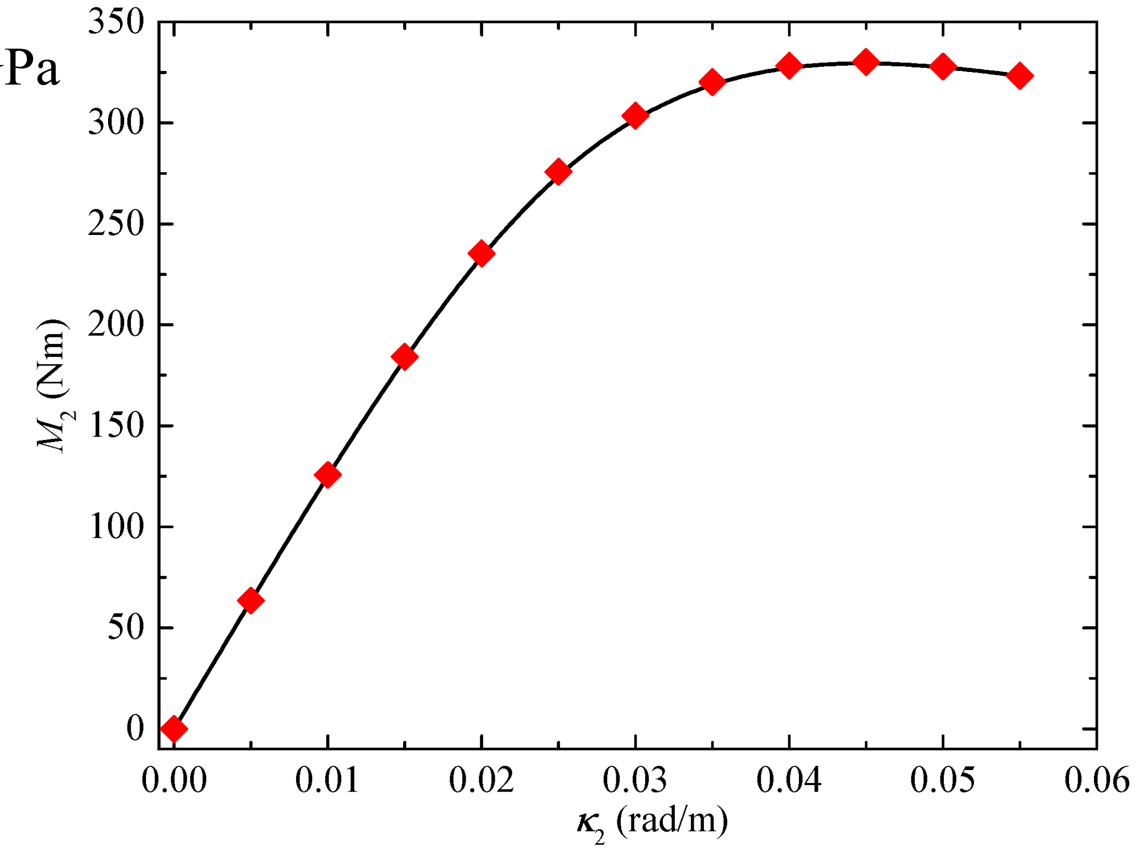
$\nu = 0.4999$

Geometry of cylinder section.

Finite Strain: Brazier Effects



Sectional ovalization of thin-walled tube section.



Nonlinear bending behavior.

Conclusion

- MSG provides a unified approach to model all beam-like structures.
- MSG theoretically achieves the best tradeoff between efficiency and accuracy.
- MSG-based beam models are proven to be much better than other existing models and
- More applications of MSG for multiscale modeling for beams should be explored.